Analyzing Electric Model-Airplane Drives

Calculation of Drive Characteristics with Spreadsheet Tools

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About

At first glance, this document may look like a scientific one, but it is not meant to be one. It is meant for the technically inclined model-airplane flier who wants to know more about the characteristics of different electric drives (and different models, for that matter) – more than he can else know without having and just trying them all in the first place. Eventually, it just describes how the calculation spreadsheets work.

In the first instance though, it defines all equations needed to represent an electric drive and derives the basic solution as prerequisites. This is meant for those seriously inclined to understand how the calculation works. Of course, *some* technical and mathematical understanding or even expertise will help but should not be required. At least there are only very simple differential equations and no integral equations, just plain algebra.

It should be even possible to skip the derivations and explanations and just go to the description of workflow and calculation tools. At least the illustrations might be interesting, though. They start with a discussion of characteristics by deriving even more equations, but it changes into showing diagrams and characteristic values for practical examples.

Many definitions, lengthy explanations and derivations may contribute to a "scientific" look. But that and the phrasing are not meant to teach the reader but just to inform him how the spreadsheet calculations have been contrived and how to interpret them.

Any personal pronoun is avoided and "we" is solely used in the sense of Pluralis Modestiae or Pluralis Auctoris (plural of modesty or author's plural, do not seem to be common in today's English), but in no case as Pluralis Majestatis (royal "we"). There should be no vowel omissions, no abbreviations, and no jargon either. However, that is all part of a quest for completeness, correctness, and conciseness.

The fonts are chosen for good on-screen readability. The pages may be displayed to fit the screen, in original size, or even enlarged – they should be easily readable in any case.

Reading this document on a display screen may be convenient because it can be searched for text strings, because there are some links to external documents in the World Wide Web, and because there are bookmarks to the chapters and sections on the left margin as well as links to pages in the text. Moreover, related illustrating diagrams are equally placed on consecutive pages so they are easily compared by toggling between these pages.

Nevertheless, this document is well suited to printing on common DIN A4 paper with a monochrome printer. For the illustrating diagrams and pictures, a color printer would be better suited, though.

Introduction

This paper explains how common spreadsheet tools like Microsoft® Office® Excel® or the free alternative LibreOffice Calc may be used to estimate the characteristics of an electric model-airplane drive. (It has been itself created and converted to PDF format with LibreOffice Writer, Math, and Draw.)

Using the word "estimate" is well-considered. No attempt is made to calculate the drive characteristics exactly or comprehensively. Quite the contrary, every possible simplification is used to define models and equations. There's nothing unusual about that since these simplifications are commonly used and their suitability is proven.

The achievable accuracy turns out to be well sufficient for the intended purpose. It's not about designing a drive optimized for a certain model but about *composing* a *suitable* drive from readily available components. These components – propeller, gear, motor, speed controller, and battery – are offered in various versions, sizes, and configurations. The point is that there are scales with certain value steps for the main features of drive components so there are only a few reasonable configurations in each case.

Propellers are offered in different kinds (sport, electric, parkflyer) as well as certain combinations of diameter and pitch. Electric motors come in certain combinations of power and specific speed (k_V). Both speed controllers and batteries just have to match the voltage (cell type and count), amperage, and capacity requirements of the chosen drive. In case a reduction gear is wanted there are usually very few choices.

Model-airplane manufacturers recommend a few reasonable motor/prop/battery combinations. And electric motor manufacturers recommend a few applications for each motor, meaning a class and weight of model as well as prop and battery. That all means that usually there are quite few choices of complete drives to be considered.

So using the word "analyzing" (electric drives) is well-considered as well. There is no way to put some desired parameters in and "calculate" the best-suited drive for a model. The only way to find it is *comparing* a few promising configurations, maybe recommended by the model manufacturer or by the motor manufacturer, or even self-chosen to give the model different characters.

The manufacturer's recommendations will usually give a typical or "mainstream" drive and model character, but for instance less power but longer flight time might be wanted, or a drive optimized for cruise flight instead of climb. Different motors and propellers are mere interchangeable modules of the whole calculation and can be checked for their suitability.

Usually, several characteristics are not exactly as specified or calculated. For instance, the field strength sample strew of the motor magnets is said to be about 10%, making for correspondingly differing k_V values. Once actual values of a real drive at hand can be measured, the whole drive calculation can be calibrated ("tweaked") to a good degree of accuracy (only a few percent error in the best case).

Since this is possible only after buying the drive, it is usually done just in edge cases, for instance before the maiden flight of a marginally powered airplane. Another important use case (which was the actual reason to develop the calculations described here) would be "building" a true-to-original simulator model. And nowadays, yet another useful application is ascertaining optimal power settings and flight speeds for cruise and climb, respectively, which can't be implemented by visual and audible impression but by appropriate telemetry in the model.

Definitions

Units

Any specification of units is enclosed in brackets [].

Dimensionless variables are marked with a null unit [-].

For convenience (no conversions), only coherent metric (SI) units are used.

Exception is rotational speed, which is specified as rpm [min^{-1}] – as usual – instead of angular speed ω in radians per second [rad/s]. Hence the conversion multiplier $2\pi/60$ is needed in some equations (2π radians per rotation, 60 seconds per minute).

No exception is made for efficiencies, which are often specified in [%] but are treated here as dimensionless ratios between 0 and 1 with a null unit [-].

Four natural unit conversions are used here, two mechanical and two electrical:

[N]=[kg·m/s^2] and [W]=[N·m/s] [A]=[V/Ω] and [W]=[V·A]

These conversions may be substituted and rearranged like ordinary equations.

Variables

R_b	[Ω]	resistance (impedance) of the battery
R_e	[Ω]	resistance (impedance) of controller (ESC), cables, and connectors
R_m	[Ω]	resistance (impedance) of the motor
R	[Ω]	resistance (impedance) of the whole system (total)
U_b	[V]	internal (no current) voltage of the battery
U_e	[V]	mean terminal voltage of the ESC as delivered to the motor
U_{mi}	[V]	mutual induction voltage of the rotating motor
I	[A]	actual current
I_{st}	[A]	stall current (rotor locked)
I_{0m}	[A]	idle current (no load) due to motor friction
I_{0g}	[A]	idle current (no load) due to gear friction
M_p	[N·m]	actual propeller moment (torque)
M_g	[N·m]	actual gear (propeller) shaft moment (torque)
M_m	[N·m]	actual motor shaft moment (external torque)
M_{st}	[N·m]	motor stall moment (total torque with rotor locked)
M_{0m}	[N·m]	motor idle moment (internal friction torque, constant)
M_{0g}	[N·m]	gear idle moment (internal friction torque, constant)
n	[s^{-1}]	actual drive speed (propeller revolutions per second)
n_p	[min^{-1}]	actual propeller speed (revolutions per minute)
n_g	[min^{-1}]	actual gear output (propeller) shaft speed
n_m	[min^{-1}]	actual gear input (motor) shaft speed
n_0	[min^{-1}]	drive idle (no-load) speed
n_{0g}	[min^{-1}]	gear shaft (drive) idle (no-load) speed
n_{0m}	[min^{-1}]	motor shaft idle (no-load) speed

P_{thrust}	[W]	actual propeller (output) thrust (propulsive) power
P_{shaft}	[W]	actual propeller (input) shaft power
P_{mech}	[W]	actual gear output (shaft) mechanical power
P_m	[W]	actual motor output (shaft) mechanical power
P_{el}	[W]	actual system input (battery) electric power
η	[-]	total system efficiency ("eta", drive and propeller)
η_m	[-]	motor efficiency (including battery and ESC)
η_g	[-]	gear efficiency
η_p	[-]	propeller efficiency
η_d	[-]	drive efficiency (motor and gear)
k_V	[min^{-1}/V]	specific rotational speed (rpm per Volt)
k_A	[A/min^{-1}]	specific current (Ampere per rpm, *negative* value)
k_M	[N·m/A]	specific moment (torque per Ampere)
i_g	[-]	gear reduction ratio (e.g. 4.4 for a 4.4:1 gear)

<u>In Mechanical-Aerodynamic Conversion and Propeller Illustration:</u>

D	[m]	propeller diameter
R	[m]	propeller radius
r	[m]	propeller local radius
c	[m]	propeller local blade chord
β	[°]	propeller local twist angle ("beta")
φ	[°]	propeller local advance angle ("phi")
α	[°]	propeller local angle-of-attack ("alpha")
H	[m]	propeller local pitch
H_n	[m]	propeller nominal pitch
v	[m/s]	flight speed (airspeed)
Δv	[m/s]	added speed (slipstream speed, see page 54)
J	[-]	propeller advance ratio
J_i	[-]	induced advance ratio (see page 55)
λ	[-]	advance ratio, alternative definition ("lambda", see page 9)
c_T	[-]	propeller thrust coefficient
c_M	[-]	propeller moment (torque) coefficient
c_P	[-]	propeller power coefficient
T	[N]	propeller thrust
η	[-]	propeller efficiency ("eta")
η^*	[-]	propulsion efficiency (see page 55)

Constants

g	[kg·m/s^2]	gravity acceleration (standard is 9.81)
ρ	[kg/m^3]	density of air ("rho", standard is 1.226)
ν	[m^2/s]	kinematic viscosity of air ("nue", standard is 0.00001464)
c	[m/s]	speed of sound in air (standard is 343.2)
π	[-]	number "pi" (3.14159)

Transformations

Specific motor moment (torque) k_M is simply specific speed k_V transformed, as well as propeller moment coefficient c_M is simply power coefficient c_P transformed:

$$k_M = \frac{60}{2 \cdot \pi \cdot k_V}$$ (see page 10)

$$c_M = \frac{c_P}{2 \cdot \pi}$$ (see page 9)

Inconsistencies

The *propeller's* rotational speed n has the unit [s^{-1}], not [min^{-1}]. That is due to the tool used for calculating the propeller coefficients, which requires this unit for the way the coefficients and the advance ratio are defined.

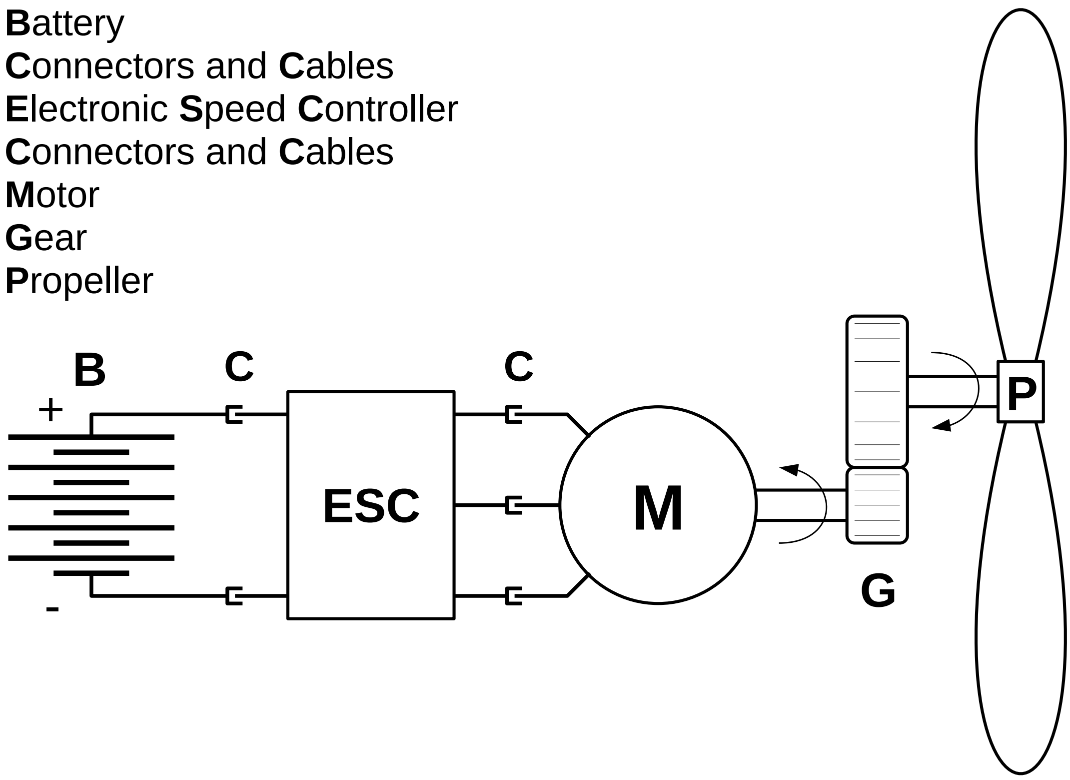
In the chapter Basic Solution, in the sections following Specific Speed/Moment, all rotational speeds are assumed to have this unit. That is not consistent with the definitions above. In the section Mechanical-Aerodynamic Conversion, n in [s^{-1}] is consistently used, though.

Simplified Drive Model

Modeling the drive means defining equations which describe it's behavior. As usual, one may discern two steps of modeling: abstraction and relaxation. In the first step – abstraction – all aspects relevant to the task are identified and all others are omitted. Usually there is still no way to draft equations, so in the second step – relaxation – even relevant aspects are omitted or at least rendered simpler than they really are. Relaxation is carried as far as necessary to find a solution in equation form.

Generic Drive Model

So we start by defining an abstract, generic model of a whole drive. The first relevant aspect is to compose the drive from common interchangeable components:



Each component is seen as a “black box” with interfaces to other components. The electrical and/or mechanical properties constitute a component’s behavior at these interfaces, which has to be described in equation form.

The *battery* may have various numbers of cells, various capacities as well as loads (C rate), and various cell types (voltages). This is simply described as particular values of corresponding variables, for instance a 5s1p 5000 mAh 30C LiPo battery.

The *ESC (electronic speed controller)* has to match the type of motor (brushed or brushless), it's size (power), and the battery’s voltage. It feeds the motor with varying electric power. For convenience, connectors and cables are assigned to the ESC.

The *motor* converts electrical to mechanical power. It may be brushed or brushless, inrunner or outrunner, and have various speeds and sizes/power.

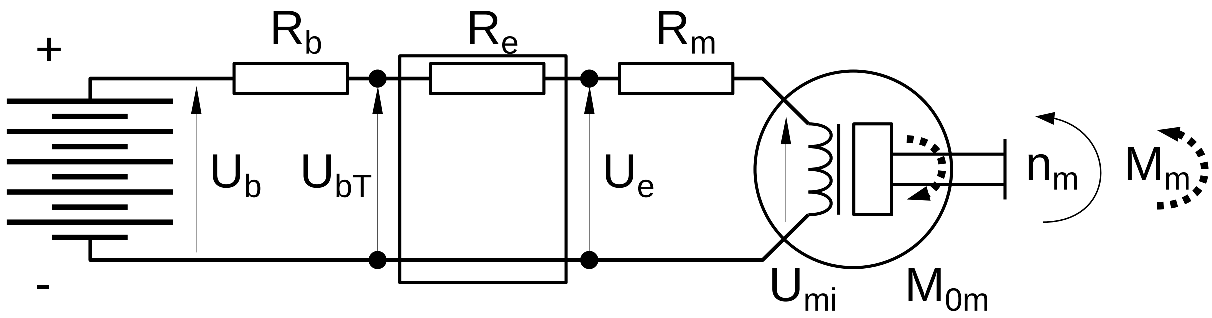
The *gear* transforms the mechanical power to different rpm and torque. It may be a spur/ring/planetary gear, and have various transmission ratios, sizes, and efficiencies.

Finally, the prop converts the mechanical power to aerodynamic thrust and torque. There are different shapes and number of blades, diameter and pitch, and folding.

Electrical-Mechanical Conversion

Electromechanical systems are modeled in the form of an equivalent circuit diagram. It specifies the system's characteristics and their interrelations.

Model-airplane motors are permanent-magnet DC motors, and a brushless motor is essentially the same, just with the mechanical commutator (collector/brushes) replaced by the ESC. (The links lead to Wikipedia.) That is why there is only one coil with two lines in the diagram's motor symbol, and why it is a DC circuit diagram. All effects of alternating and pulsing current are neglected or replaced by resistances, respectively. That is a very useful and still acceptable simplification.



The battery provides an "internal" voltage, which depends on type and number of cells. We use the standard or nominal voltage of the cell type at hand, that is 3.7 V for LiPo, 3.3 V for LiFePo, and 1.2 V for NiMH or NiCd. According to Ohm's law, the battery's terminal voltage is lower than the internal voltage while any current is flowing because there is some (complex) internal impedance in a battery, here replaced by a simple (constant) resistor:

$$U_{bT} = U_b - R_b \cdot I$$

The ESC reduces the voltage as well due to its internal resistance, which includes all connector and cable resistances in our simplified model. Beyond that, its "throttle" function is seen here simply as further reduction of (mean) voltage. At WOT (wide open throttle), the ESC delivers a slightly reduced voltage to the motor:

$$U_e = U_{bT} - R_e \cdot I$$

Of course, also the motor has an ohmic resistance, which is a replacement for the real ohmic resistance as well as for complex electric and magnetic impedances. In the simplified model, the motor coil sees a slightly reduced voltage:

$$U_{mC} = U_e - R_m \cdot I$$

Further simplifying our model, we assume that the ESC's "throttle" function reduces *the battery's* internal voltage. That allows to sum up one single ohmic resistance:

$$U_{mC} = U_b - (R_b + R_e + R_m) \cdot I = U_b - R \cdot I$$

This voltage applied to the motor coil is antagonized by an opposing voltage that is literally generated in the spinning motor by so-called mutual induction and sometimes also aptly called generator voltage. It is proportional to rotational speed and here is where the k_V value (*specific* rotational speed) comes into play:

$$U_{mi} = n_m / k_V \quad \text{and} \quad U_{mC} = U_{mi} \quad \text{hence} \quad k_V = n_m / U_{mi} \quad \text{or} \quad k_V = n_m / U_{mC}$$

This equation shows that k_V essentially tells how fast the motor spins proportional to the effective voltage applied. That is one of the main motor characteristics, depending on number of poles, number of windings, and the motor's geometry/size.

Disregarding any current, the voltage effective in the motor is the battery voltage reduced by the mutual-induction voltage:

$$U_{eff} = U_b - U_{mi} = U_b - n_m/k_V$$

Then, according to Ohm's law, the current flowing through the motor coil and the whole system is the ratio of effective voltage and total resistance:

$$I = U_{eff}/R = (U_b - n_m/k_V)/R$$

If the motor is stalled (blocked), there is no rotation, hence no mutual induction, and current depends solely on ohmic resistance:

$$I_{st} = U_b/R$$

For convenience, we will use current (amperage) expressed as directly dependent on rotational speed. That is possible by means of *specific* current k_A, which (as a *negative* value) tells how much current flows *inversely* proportional to rotational speed:

$$I = I_{st} + k_A \cdot n_m \quad \text{what makes} \quad k_A = \frac{-1}{R \cdot k_V} \quad (I \text{ and } I_{st} \text{ substituted})$$

k_V can be transformed into specific moment (torque) k_M, which directly (hence conveniently) tells how much moment (torque) is *produced* proportional to current:

$$M_m = I \cdot k_M$$

However, this equation is provisional. In addition to the electric losses, there are complex mechanic and magnetic losses in the motor. For simplicity's sake, they are represented by a constant internal friction moment that *re*duces the torque output:

$$M_m = I \cdot k_M - M_{0m}$$

An idling motor doesn't produce any output torque, but it still has to overcome the internal friction moment what requires a corresponding idle current:

$$M_{0m} = I_{0m} \cdot k_M \quad \text{hence} \quad I_{0m} = M_{0m}/k_M \quad \text{and} \quad M_m = (I - I_{0m}) \cdot k_M$$

Even in case of stall (blocked rotor) this friction (idle) moment is assumed active. Yet for convenience we *define* the stall moment as total torque produced internally:

$$M_{st} = I_{st} \cdot k_M$$

By the way, sometimes the friction moment is omitted in drive calculations. That spoils the calculation, which is actually simple: voltage makes for speed (rpm), and amperage makes for moment (torque), both proportionally and interdependently.

However, now the motor's output (mechanical) power can be derived from torque and speed, and its input (electrical) power from current and (battery) voltage. Additionally, the simple terms are substituted with complex ones which contain only specified constants and the rotational speed as sole variable, just to demonstrate that:

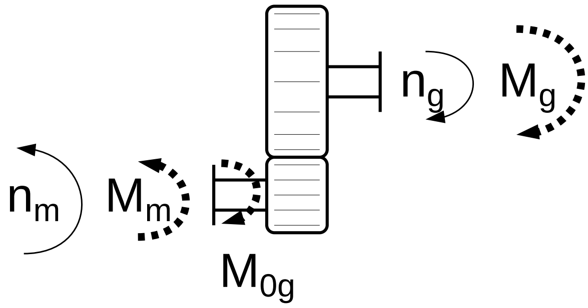
$$P_m = M_m \cdot \frac{2 \cdot \pi}{60} \cdot n_m = \frac{U_b - R \cdot I_{0m}}{R} \cdot \frac{n_m}{k_V} - \frac{1}{R} \cdot \left(\frac{n_m}{k_V}\right)^2 \qquad P_{el} = I \cdot U_b = \frac{U_b^2}{R} - \frac{U_b}{R} \cdot \frac{n_m}{k_V}$$

Because the system's total resistance was used in the calculations so far, this resistance is included in the efficiency as well. So motor efficiency actually means drive efficiency, still not including the gear (whose efficiency is later included by multiplication). The drive's efficiency is (provisionally) just the ratio of motor powers out/in:

$$\eta_m = \frac{P_m}{P_{el}} = \frac{(U_b - R \cdot I_{0m}) \cdot k_V \cdot n_m - n_m^2}{U_b^2 \cdot k_V^2 - U_b \cdot k_V \cdot n_m}$$

Mechanical-Mechanical Conversion

Mechanical systems are modeled in the form of a schematic sketch or, more specifically, a free-body diagram. It shows several connected bodies or a single body with all of their applied forces and moments, that is their "interface". The gear is one of the bodies and is a simple transmission or rpm/torque transformer, respectively, between motor and propeller.



The most obvious gear property is its transmission ratio. In the case of model-airplane drives, it is always a reduction ratio. That means a quite high motor speed is reduced to a lower propeller speed. Conversely, a quite low motor moment is transformed to a higher propeller torque. Both speed and torque *directions* may be reversed – by a spur gear like in the sketch above – but that does not matter in our calculations. However, the second equation is provisional:

$$n_g = n_m / i_g \qquad M_g = M_m \cdot i_g$$

After all, the gear makes for some power losses. Obviously, rotational speeds are mechanically fixed so the losses appear as reduction of torque. That is plausible since the losses stem from friction. We can see this in two extremely simple ways: constant or proportionally dependent on moment (torque). Either way, the moments are reduced and we just assume (define) it is the input moment:

$$M_g = (M_m - M_{0g}) \cdot i_g$$

Difference is that *in this first case* the friction moment as an absolute value has to be measured or guessed, what may be hard. Easier may be just estimating and later "tweaking" a gear efficiency as a relative value (*second case*):

$$M_{0g} = (1 - \eta_g) \cdot M_m \quad \text{or directly} \quad M_g = M_m \cdot \eta_g \cdot i_g$$

Probably a combination of both ways (and even non-proportional and speed-dependent friction) would be correct, but for simplicity's sake one of the two ways is chosen. *In any case*, gear friction can be treated like internal motor friction so a gear friction moment requires a corresponding current just like the motor friction moment.

That solves the problem of calculating a gear efficiency *in the first case* by calculating a total mechanical drive power and efficiency:

$$I_{0g} = M_{0g} / k_M \qquad \text{makes}$$

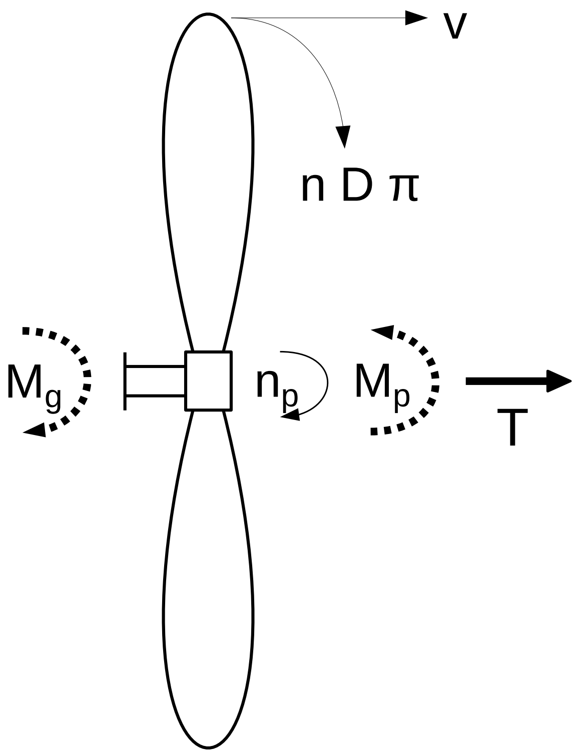
$$P_{mech} = M_g \cdot \frac{2 \cdot \pi}{60} \cdot n_g = \frac{U_b - R \cdot (I_{0m} + I_{0g})}{R} \cdot \frac{i_g}{k_V} \cdot n_g - \frac{1}{R} \cdot \left(\frac{i_g}{k_V} \right)^2 \cdot n_g^2$$

Again *in any case*, the drive's efficiency is finally the ratio of mechanical power output to the gear shaft and electrical power input from the battery:

$$\eta_d = \frac{P_{mech}}{P_{el}} \quad \text{or else in our } \textit{second case}\text{:} \quad \eta_d = \eta_m \cdot \eta_g$$

Mechanical-Aerodynamic Conversion

The next and last body, the propeller, is a complicated component aerodynamically, so we will have to rely on a correspondingly complex, specialized tool to calculate the moment (torque) and other coefficients.



Propellers are usually characterized by dimensionless coefficients seen as valid for a certain geometric shape, regardless of size and speed. They are in some way related to the propeller's diameter D as a common measure of size, as well as to rotational speed n [s^{-1}]. There are three characteristics (T, M, P) and corresponding coefficients (c_T, c_M, c_P):

Thrust $$T = c_T \cdot \rho \cdot n^2 \cdot D^4$$

Moment (torque) $$M_p = c_M \cdot \rho \cdot n^2 \cdot D^5$$

Power $$P_{shaft} = c_P \cdot \rho \cdot n^3 \cdot D^5$$

Now shaft power is also:

$$P_{shaft} = 2 \cdot \pi \cdot n \cdot M_p = 2 \cdot \pi \cdot c_M \cdot \rho \cdot n^3 \cdot D^5$$

what makes $$c_M = \frac{c_P}{2 \cdot \pi}$$

In some way, these characteristics also depend on flight speed. So the coefficients, as dimensionless values, must be related to a kind of dimensionless flight speed as well as rotational speed. That is the advance ratio, which is meant to be the ratio of flight speed v [m/s] and circumferential blade-tip speed:

$$\lambda = \frac{v}{n \cdot D \cdot \pi}$$ but actually used is this slightly simpler definition: $$J = \frac{v}{n \cdot D}$$

Anyway, the tool mentioned above delivers the coefficients over the whole range of advance ratios, or from zero speed to top speed, as it were. So it is possible to calculate the mechanical power needed to spin the propeller (shaft power, equation above) and the thrust power (also called propulsive power) produced by it:

$$P_{thrust} = T \cdot v = J \cdot c_T \cdot \rho \cdot n^3 \cdot D^5$$ because $$v = J \cdot n \cdot D$$

The propeller's efficiency is the ratio of these two powers:

$$\eta_p = \frac{P_{thrust}}{P_{shaft}}$$ what makes it directly (dimensionless): $$\eta_p = J \cdot \frac{c_T}{c_P}$$

Total system efficiency is the ratio of thrust power and electrical power:

$$\eta = \frac{P_{thrust}}{P_{el}}$$ or else: $$\eta = \eta_m \cdot \eta_g \cdot \eta_p$$

See Martin Hepperle's JavaProp Users Guide.

Basic Solution

Approach

All the equations presented above show that a drive's behavior can be described as dependent on several given constants and one single variable – *rotational* speed. That includes even the propeller, so eventually the drive's behavior can be described over a whole *flight* speed range from "static" (zero speed) to "pitch speed" (zero thrust).

Of course, this was intended since it is the basis for a solution in equation form. There has to be – and now can be – one single equation that delivers the rotational speed of the whole drive including propeller. To develop this equation, we have to equate something of the drive with the same thing of the propeller.

We want to equate the motor/gear (drive) torque with the propeller torque (moment), both dependent on rotational speed. We prefer the *torques* to get only a second-order polynomial. Equating motor/gear and propeller *powers* would give a third-order polynomial, which would be unnecessarily complicated.

Accordingly, the term "constant" means independent of rpm; "decrease" or "increase" mean change with rpm or even rpm squared, respectively.

In the following derivation, the given k_V value is not used but the k_M and k_A values instead because that is more convenient. As mentioned above and proven below, k_M is k_V transformed, and k_A is easily calculated from two given constants.

There are those two extremely simple ways to see gear losses: constant, or proportional to torque. The former makes for simpler and more obvious combined constants while the latter seems to be more practical. Both ways are presented here for comparison, but we will stick to the more practical way after that.

We get only a *basic* solution in the end insofar as it just gives rotational speed dependent on propeller power coefficient. However, specialized propeller analysis tools deliver this and other coefficients for the whole range of possible advance ratios. That is equivalent to flight speed range, so deriving all other characteristics for this range is possible then.

Specific Speed/Moment

The transformation of specific speed k_V into specific moment (torque) k_M has not been derived yet. It may be appropriate to make up for that before using it in the solution.

We consider only the conversion of electrical into mechanical power "inside" the motor but omit (disregard) electrical losses "before" and mechanical losses "after" it. The constants define "internal" voltage and moment, respectively. Multiplying them by current and speed, respectively, yields "internal" powers. Equating mechanical with electrical power clearly shows the transformation in question (Wikipedia).

$$U_{mi} = \frac{n_m}{k_V} \qquad P_{el} = U_{mi} \cdot I = \frac{n_m}{k_V} \cdot I$$

$$M_m = I \cdot k_M \qquad P_{mech} = M_m \cdot \frac{2 \cdot \pi}{60} \cdot n_m = I \cdot k_M \cdot \frac{2 \cdot \pi}{60} \cdot n_m$$

$$P_{mech} = P_{el} \qquad I \cdot k_M \cdot \frac{2 \cdot \pi}{60} \cdot n_m = \frac{n_m}{k_V} \cdot I \qquad k_M = \frac{60}{2 \cdot \pi \cdot k_V}$$

Drive Torque 1

Quite simple and obvious combined constants K_1 and K_2 result for the drive (motor and gear) from assuming constant gear friction (*first case* above).

Torque comes from current in the electric motor, so this to begin with:

$$I = I_{st} + k_A \cdot n_m = I_{st} + k_A \cdot i_g \cdot n_g \qquad \text{because } n_m = i_g \cdot n_g$$

That makes the drive's torque (moment) dependent on rotational speed (rpm):

$$M_g = k_M \cdot I - M_{0m} - M_{0g} = (I - I_{0m} - I_{0g}) \cdot k_M \qquad \text{because } M_{0m/g} = k_M \cdot I_{0m/g}$$

$$M_g = (I_{st} - I_{0m} - I_{0g} + k_A \cdot i_g \cdot n_g) \cdot k_M \qquad \text{I substituted with equation above}$$

$$M_g = I_{st} \cdot k_M - I_{0m} \cdot k_M - I_{0g} \cdot k_M + k_A \cdot k_M \cdot i_g \cdot n_g \qquad \text{expanded}$$

Combining the constants makes things clearly arranged:

$$K_1 = (I_{st} - I_{0m} - I_{0g}) \cdot k_M \qquad \text{torque output when stalled [Nm]}$$

$$K_2 = 60 \cdot k_A \cdot k_M \cdot i_g \qquad \text{torque decrease with speed [Nm/s}^{-1}\text{]}$$

$$M_g = K_2 \cdot n_g + K_1 \qquad \text{speed } n_g \text{ in rotations per } \textit{second} \text{ [s}^{-1}\text{]}$$

Drive Torque 2

Assuming gear friction in the form of gear efficiency, that is proportional to moment and hence inversely proportional to rotational speed (*second case* above), results in slightly more complicated combined drive constants K_1 and K_2. Yet it is a more practicable way than the first one, and it is the usual way.

Torque comes from current in the electric motor, so this to begin with:

$$I = I_{st} + k_A \cdot n_m = I_{st} + k_A \cdot i_g \cdot n_g \qquad \text{because } n_m = i_g \cdot n_g$$

That makes the motor torque (moment) dependent on rotational speed (rpm):

$$M_m = k_M \cdot I - M_{0m} = (I - I_{0m}) \cdot k_M \qquad \text{because } M_{0m} = k_M \cdot I_{0m}$$

$$M_m = (I_{st} - I_{0m} + k_A \cdot i_g \cdot n_g) \cdot k_M \qquad \text{I substituted with equation above}$$

Introduce gear efficiency and expand the equation:

$$M_g = M_m \cdot \eta_g \cdot i_g$$

$$M_g = (I_{st} - I_{0m} + k_A \cdot i_g \cdot n_g) \cdot k_M \cdot \eta_g \cdot i_g \qquad M_m \text{ substituted with equation above}$$

$$M_g = I_{st} \cdot k_M \cdot \eta_g \cdot i_g - I_{0m} \cdot k_M \cdot \eta_g \cdot i_g + k_A \cdot k_M \cdot \eta_g \cdot i_g^2 \cdot n_g \qquad \text{expanded}$$

Combining the constants makes things clearly arranged:

$$K_1 = (I_{st} - I_{0m}) \cdot k_M \cdot \eta_g \cdot i_g \qquad \text{torque output when stalled [Nm]}$$

$$K_2 = 60 \cdot k_A \cdot k_M \cdot \eta_g \cdot i_g^2 \qquad \text{torque decrease with speed [Nm/s}^{-1}\text{]}$$

$$M_g = K_2 \cdot n_g + K_1 \qquad \text{speed } n_g \text{ in rotations per } \textit{second} \text{ [s}^{-1}\text{]}$$

Propeller Torque

Propeller moment (torque) depends on rotational speed (rps) in any case:

$$M_p = c_M \cdot \rho \cdot D^5 \cdot n_p^2$$ speed n_p in rotations per *second* [s^{-1}]

Again combining the constants makes:

$$K_3 = \rho \cdot D^5$$ torque increase with speed [Nm/s^{-2}]

$$M_p = c_M \cdot K_3 \cdot n_p^2$$ c_M is not constant!

Formal Solution

Equating propeller torque with drive torque is our approach:

$M_p = M_g = M$ (and of course $n_p = n_g = n$)

This is really clearly arranged and easy:

$$c_M \cdot K_3 \cdot n_p^2 = K_2 \cdot n_g + K_1$$ substituted

$$c_M \cdot K_3 \cdot n^2 + (-K_2) \cdot n + (-K_1) = 0$$ rearranged (normalized)

There are standard solutions for such a second-order polynomial:

$$\Delta = 4 c_M K_3 (-K_1) - (-K_2)^2$$ discriminant

The discriminant is negative for all values of c_M, so there are two possible solutions:

$$n_{1,2} = \frac{-(-K_2) \pm \sqrt{(-K_2)^2 - 4 c_M K_3 (-K_1)}}{2 c_M K_3}$$ possible solutions

n_1 (with positive square root) is the correct solution since n_2 would be negative.

That was the basic solution's formal derivation, but we can write it simpler:

$$n = \frac{K_2 + \sqrt{K_2^2 + 4 c_M K_3 K_1}}{2 c_M K_3}$$ solution

See Quadratic Formula at Wikipedia. Dimensional analysis is done in the next section.

Applicable Solution

Finally, we want to make the polynomial constants K_1, K_2, and K_3 directly and exclusively depend on given constants, which usually are U_b, R, I_{0m}, k_V, i_g, η_g, ρ, D, and c_P. That also subtly modifies the second-order polynomial they belong to:

Substituting c_M with c_P makes

$$c_M \cdot K_3 \cdot n^2 + (-K_2) \cdot n + (-K_1) = 0 \quad \rightarrow \quad c_P \cdot K_3 \cdot n^2 - K_2 \cdot n - K_1 = 0 \quad \text{[Nm]}$$

K_1 is the polynomial's constant term, and since we equated drive and propeller moment, it must be a moment as well. In fact, it is the drive's torque output when stalled, is a fixed positive number, and has the unit [Nm]:

Substituting $I_{st} = \frac{U_b}{R}$ and $k_M = \frac{60}{2 \cdot \pi \cdot k_V}$ makes

$$K_1 = (I_{st} - I_{0m}) \cdot k_M \cdot \eta_g \cdot i_g \quad \rightarrow \quad K_1 = \frac{60}{2 \cdot \pi} \cdot \left(\frac{U_b}{R} - I_{0m} \right) \cdot \frac{i_g}{k_V} \cdot \eta_g \quad \text{[Nm]}$$

K_2 is the term proportional to rotational speed n [s^{-1}], so it must be moment change. In fact, it is the drive's torque-output decrease and therefore a negative number. Because rotational speed is rotations *per second* here, the unit is [Nm/s^{-1}]:

Substituting $k_A = \frac{-1}{k_V \cdot R}$ and $k_M = \frac{60}{2 \cdot \pi \cdot k_V}$ in the K_2 equation and expanding *the formal solution* (above) with ½ makes

$$K_2 = 60 \cdot k_A \cdot k_M \cdot \eta_g \cdot i_g^2 \quad \rightarrow \quad K_2 = -\frac{900}{\pi} \cdot \frac{1}{R} \cdot \left(\frac{i_g}{k_V} \right)^2 \cdot \eta_g \quad \text{[Nm/s}^{-1}\text{]}$$

K_3 is the term proportional to rotational speed squared n^2 [s^{-2}], so it is a moment change as well. It is the propeller's torque-input increase and therefore a positive number. It comes from aerodynamic lift and drag on the propeller blades, what lets it increase with airspeed squared and hence rotational speed squared, so the unit has to be [Nm/s^{-2}]:

Substituting $c_M = \frac{c_P}{2 \cdot \pi}$ *in the polynomial* (above) makes

$$K_3 = \rho \cdot D^5 \quad \rightarrow \quad K_3 = \frac{\rho \cdot D^5}{2 \cdot \pi} \quad \text{[Nm/s}^{-2}\text{]}$$

Now we can write the solution in a form more practical for use:

$$n = \frac{K_2 + \sqrt{K_2^2 + 4 c_M K_3 K_1}}{2 c_M K_3} \quad \rightarrow \quad n = \frac{K_2 + \sqrt{K_2^2 + c_P K_3 K_1}}{c_P K_3} \quad \text{[s}^{-1}\text{]}$$

Motor/Gear Illustration

Basic Characteristics

As a first step, we infer and interpret some basic drive characteristics, which will become apparent in the diagrams later in this chapter. We draw on equations derived for the basic solution in the previous chapter, primarily the drive's moment/torque output:

$M_g = K_2 \cdot n_g + K_1$ [Nm] speed n_g here in rotations per *minute* [min^{-1}]

The constant K_1 is the drive's torque output when stalled, that is at zero speed.

$$K_1 = \left(\frac{U_b}{R} - I_{0m} \right) \cdot \frac{60}{2 \cdot \pi} \cdot \frac{i_g}{k_V} \cdot \eta_g \quad \text{[Nm]}$$

This is a measure of the drive's power, or more specifically of its *torque-output power*. It is just worth noting which properties make for a big torque. The constant's equation has three parts or terms:

The term in parentheses is the maximum current (amperage) that the drive can draw and turn into motor torque. Powerful motors employ high battery voltage and/or thick-wire windings with low resistance, which is also one measure of the motor's quality. Motors of the same size may have more or less electric resistance, giving more or less power. (And we include the battery's and the ESC's resistance – size and quality – in the drive's resistance.) Another quality measure is low internal mechanical motor friction, which has to be overcome by a corresponding idle current and which usually increases with motor size. Still, the bigger and "better" a motor is, the more torque-output power it has.

The following term's first part is the unit conversion multiplier that is needed because we specify specific speed k_V as rotational speed (rpm) instead of angular speed. In the term's second part, the motor's specific speed k_V – rpm per Volt – is inverted. Together with the conversion multiplier, that is actually the motor's specific moment k_M – torque per Ampere. k_V is in the denominator, so the faster the motor can spin (high k_V) the less torque it can produce (low k_M). These are relative (specific) values, and in addition it is true that an absolutely bigger motor tends to spin slower and produce more torque, that is its k_V value is lower and its k_M value is higher, respectively.

The motor's specific speed k_V divided by gear reduction ratio i_g is the whole drive's specific speed. Its inverse value, like in this equation, is the whole drive's specific moment. The gear reduction ratio i_g is the factor by which the motor's torque is increased and the motor's rotational speed is decreased. A gear is used for relatively slow-spinning, relatively big – that is lightly loaded – propellers for which a matching motor alone would be too big, heavy, and powerful. The reduction gear actually transforms the motor's specific speed and moment to lower and higher drive values, respectively.

The equation's last term is simply gear efficiency. It is the proportion of motor torque-output transmitted by the gear, and is a measure of the gear's quality. Bigger gears, which are needed for more torque, tend to be more efficient.

Obviously, these relations are all proportional: Torque is proportional to voltage, inversely to resistance, just friction subtracted; it is proportional to specific moment (so inversely to specific speed), gear efficiency, and reduction ratio. The bigger the drive the more torque. That is true at any rotational speed and especially at zero speed. So there will be a straight line in a diagram showing torque over speed, and the constant K_1 is the torque-axis intercept of this line – if the torque axis is located at the zero-speed point of the speed axis, that is (see section Motor/Gear Diagrams).

Then again, the constant K_2 is the line's slope, which is negative, meaning torque-output decreases when rotational speed increases.

$$K_2 = -\frac{1}{R} \cdot \frac{60}{2\cdot\pi} \cdot \left(\frac{i_g}{k_V}\right)^2 \cdot \eta_g \quad [\mathrm{Nm/min^{-1}}]$$ (In this form *not* applicable to *solution*!)

This is a measure of the drive's "rigidity" or *elasticity*, respectively, it's decrease of rotational speed with increase of load. This constant's equation has three parts or terms, too:

The first term is just the drive's resistance inverted. In this case, it also makes the constant K_2 negative. High resistance makes for a low line slope, meaning a quite elastic drive. If such a drive's torque load is increased, rotational speed will be noticeably reduced. Conversely, a "better" drive with lower resistance is more "rigid". On the other hand, resistance is not only a quality measure. The bigger a motor's size and the higher its specific speed k_V the lower tends to be the whole drive's resistance.

Again, the second term's first part is the required unit conversion multiplier for rotational speed. The second part is again the whole drive's specific speed inverted, but it is even squared now. Actually, the unit conversion multiplier times the first part of this square is specific moment and the second part is specific speed inverted. A low-torque *and* – hence – fast-spinning drive is quite elastic, and vice versa. That is the reason why this relation is quadratic.

Put another way: K_1 as the torque-line's torque-axis intercept goes up with bigger drive specific moment, while idle speed n_0 as the speed-axis intercept goes down with smaller drive specific speed (see next section). So both *together* make for a steeper torque line what is expressed by the drive's specific speed inverted *squared*. Since a big and/or geared drive spins slow *and* has high torque, it is *over*-proportionally more "rigid" than a small and/or direct drive, which spins fast *and* has low torque, and is quite elastic.

The equation's last term is again gear efficiency. The "better" and bigger the gear, the bigger is the proportion of motor torque that is transformed to drive torque-output and the more "rigid" is the drive. A "cheaper" and smaller gear gives a more elastic drive.

In practice, not only η_g but also the values of R, I_{0m}, k_V, and even i_g are often *not accurately* known. But if some real currents and rotational speeds are known by measurement, the drive calculation may be possibly calibrated ("tweaked") by varying R. Usually a value can be found that makes the calculated currents and speeds equal to the measured ones by "correcting" both constants K_1 and K_2 at the same time. A practical value of η_g can be found in the calibration process as well.

Characteristic Speeds and Quantities

To illustrate the motor/gear combination's characteristics dependent on rotational speed, we need equations for some *characteristic* rotational speeds as well. And for some of them, equations for the corresponding quantities have to be derived.

The rotational speed of a stalled motor is zero by definition:

$$n_{st} = 0$$

Maximum rotational speed is "theoretical" or "ideal" because it could be reached only if there were no friction. It is where current I is (or would be) zero. We use two equations from the Electrical-Mechanical Conversion section to substitute variables in a current equation from the Drive Torque 2 section. Equating this with zero gives maximum rotational speed $n_{g\,max}$:

$$I_{st} = \frac{U_b}{R} \qquad k_A = \frac{-1}{R\cdot k_V} \qquad I = I_{st} + k_A\cdot i_g\cdot n_g = \frac{U_b}{R} - \frac{i_g\cdot n_g}{R\cdot k_V} = 0 \qquad n_{g\,max} = U_b\cdot\frac{k_V}{i_g}$$

The point (rotational speed) where the moment (torque) output is zero, is called idle (or no-load) speed n_0. In the section Electrical-Mechanical Conversion, we had an equation for I dependent on rotational speed. When idle, current flows only to overcome internal motor friction, that is idle current I_{0m}. By definition, there is no gear friction when idle (no moment output) in our *second case* (gear efficiency η_g is specified). So just equating current I with motor idle current I_{0m} gives idle speed n_0:

$$I = \left(U_b - \frac{i_g\cdot n_g}{k_V}\right)\cdot\frac{1}{R} = I_{0m} \quad \text{rearranged results in} \quad n_0 = n_{0g} = \left(U_b - R\cdot I_{0m}\right)\cdot\frac{k_V}{i_g}$$

Next is the point of maximum mechanical power output P_{mech}. There is a maximum because P_{mech} is a negative (inverted) parabola as shown in Mechanical-Mechanical Conversion. Here we write the equation in the form for our *second case*. Then we differentiate P_{mech} with respect to n_g and equate the result with zero. That reveals the *position* of maximum mechanical power output being at half idle speed:

$$P_{mech} = M_g\cdot\frac{2\cdot\pi}{60}\cdot n_g = M_m\cdot\eta_g\cdot i_g\cdot\frac{2\cdot\pi}{60}\cdot n_g = -\frac{\eta_g\cdot i_g^2}{R\cdot k_V^2}\cdot n_g^2 + \left(U_b - R\cdot I_{0m}\right)\cdot\frac{\eta_g\cdot i_g}{R\cdot k_V}\cdot n_g$$

$$\frac{dP_{mech}}{dn_g} = -\frac{2\cdot\eta_g\cdot i_g^2}{R\cdot k_V^2}\cdot n_g + \left(U_b - R\cdot I_{0m}\right)\cdot\frac{\eta_g\cdot i_g}{R\cdot k_V} = 0 \qquad n_{g\,Pmax} = \frac{U_b - R\cdot I_{0m}}{2}\cdot\frac{k_V}{i_g} = \frac{n_0}{2}$$

In the equation for mechanical power, drive speed is substituted with the equation for drive speed at maximum power. That gives the *value* of maximum mechanical power:

$$P_{mech\,max} = \frac{\left(U_b - R\cdot I_{0m}\right)^2}{4\cdot R}\cdot\eta_g$$

Because P_{mech} is an inverted parabola, drive efficiency η_d is one as well, just skewed by the inversely proportional P_{el} line, so there is a maximum as well. This one is significantly harder to derive as equation. We already saw that when we substituted the P terms with more complicated expressions dependent on drive rotational speed n_g. The usual trick is using expressions dependent on current I and finally substituting this with n_g. So first we make the two powers and their components depend on current.

For that we need an equation giving rotational speed n_g dependent on current I. We equate two equations from the section Electrical-Mechanical Conversion and substitute with one from the section Mechanical-Mechanical Conversion to derive this equation:

$$U_{mC} = U_b - R\cdot I \quad \text{and} \quad U_{mC} = \frac{n_m}{k_V} \quad \text{and} \quad n_g = \frac{n_m}{i_g} \quad \text{make} \quad n_g = (U_b - R\cdot I)\cdot\frac{k_V}{i_g}$$

And we need an equation giving motor moment M_m dependent on current I. We use two equations from the Basic Solution chapter:

$$M_m = (I - I_{0m})\cdot k_M \quad \text{and} \quad k_M = \frac{60}{2\cdot\pi\cdot k_V} \quad \text{make} \quad M_m = (I - I_{0m})\cdot\frac{60}{2\cdot\pi\cdot k_V}$$

Now substituting M_m and n_g in the P_{mech} equation above gives the needed equation. It is just motor power output P_m expressed in electrical terms, being the proportion of current producing moment output times the proportion of voltage producing rotational speed. Gear efficiency reduces moment and hence also power output:

$$P_{mech} = M_m\cdot\eta_g\cdot i_g\cdot\frac{2\cdot\pi}{60}\cdot n_g = (I - I_{0m})\cdot(U_b - R\cdot I)\cdot\eta_g = P_m\cdot\eta_g$$

Electrical power input P_{el} depends on current I, anyway:

$$P_{el} = U_b\cdot I$$

The equation for P_{mech} above showed (again) that we can substitute it with motor power and this way get a simpler equation for drive efficiency. As desired, substituting the powers with the equations above makes for a manageable efficiency equation. Then we differentiate η_d with respect to I and equate the result with zero. That shows gear efficiency having no influence on *the point of* maximum drive efficiency:

$$\eta_d = \frac{P_{mech}}{P_{el}} = \frac{P_m}{P_{el}}\cdot\eta_g = \frac{(I - I_{0m})\cdot(U_b - R\cdot I)}{I\cdot U_b}\cdot\eta_g = \left(1 - \frac{R\cdot I}{U_b} - \frac{I_{0m}}{I} + \frac{R\cdot I_{0m}}{U_b}\right)\cdot\eta_g$$

$$\frac{d\,\eta_d}{d\,I} = -\frac{R}{U_b}\cdot\eta_g + I_{0m}\cdot\eta_g\cdot\frac{1}{I^2} = 0 \quad \text{gives} \quad I_{\eta\max} = \sqrt{\frac{U_b\cdot I_{0m}}{R}}$$

Using the drive speed n_g equation above and substituting current I with this square-root equation, finally results in the *point* (rotational speed) of maximum drive efficiency. In the drive efficiency equation, current I is substituted with the equation for current at maximum efficiency, giving maximum drive efficiency's *value*. Rearranging the equation after substitution is not simple, yet a quite short equation results:

$$n_{g\,\eta\max} = \left(U_b - \sqrt{U_b\cdot R\cdot I_{0m}}\right)\cdot\frac{k_V}{i_g} \quad \text{is position and value is} \quad \eta_{d\max} = \left(1 - \sqrt{\frac{R\cdot I_{0m}}{U_b}}\right)^2\cdot\eta_g$$

See Feature Article by Joachim Bergmeyer and his derivations.

Characteristic-Speed Ratios

Now that we have those characteristic speeds we can relate them to each other. There are no surprises, just a few insights that might be useful for assessing drives.

First, relating idle speed to "theoretical" or "ideal" maximum speed shows what both kinds of losses mean for a motor and for a motor-gear combination (drive) as well:

$$\frac{n_0}{n_{max}} = \frac{U_b - R \cdot I_{0\,m}}{U_b} = 1 - \frac{R \cdot I_{0\,m}}{U_b}$$

After all, system impedance R represents all electrical losses and idle current I_{0m} all mechanical losses. Of course this is simplified, and by definition there are no gear losses in this *second case* where gear friction is proportional to moment output, which is zero here.

Impedance R times idle current I_{0m} is the voltage drop when idling. Relating that to battery voltage U_b is the proportion of this battery voltage lost. That subtracted from 1 is the proportion of battery voltage left and seen by the motor coil. Now since voltage makes for rotational speed (proportionate to the k_V value), that is also the proportion of "theoretical" or "ideal" maximum speed remaining in reality as idle speed.

A "better" motor means less electrical and mechanical losses than those of a "cheap" motor. This is achieved for instance by using neodymium magnets instead of ferrite magnets, ball bearings instead of sleeve bearings, and better collector and brushes in case of a brushed motor or better ESC in case of a brushless, respectively. The better a motor is, the closer is its idle speed to the "theoretical" or *ideal* maximum speed.

So all losses in a drive result in more or less reduction of rotational speed. What we have seen for idle-speed so far will hold for other characteristic speeds as well. Now this idle-speed will be used as practical reference for more ratios, which will be just a bit more complicated, though.

In the previous section, we had already seen that maximum mechanical power output $P_{mech\ max}$ is delivered at half idle-speed. The derivation is repeated here, just to show that the idle/ideal speed ratio is contained twice:

$$\frac{n_{Pmax}}{n_0} = \frac{\frac{U_b - R \cdot I_{0\,m}}{2}}{U_b - R \cdot I_{0\,m}} = \frac{\frac{1}{2} \cdot \left(1 - \frac{R \cdot I_{0\,m}}{U_b}\right)}{1 - \frac{R \cdot I_{0m}}{U_b}} = \frac{\frac{1}{2} \cdot \frac{n_0}{n_{max}}}{\frac{n_0}{n_{max}}} = \frac{1}{2}$$

Actually this is simple and general: Half idle-speed is the lowest speed that is reasonable by all means. It is because at lower speeds, the mechanical power output is lower while the electrical power input is even higher, and that is inefficient.

In practice, even this speed may be too low. Efficiency is not exactly good there, and that means a lot of heat is produced in the motor. Depending on power setting (voltage) and heat removal (cooling), even short-time tolerable power may be lower than maximum power and thus tolerable speed higher than maximum-power speed. In that case, half idle-speed is a "theoretical" lower limit, but it is still the *absolute* lower limit by all means.

The practical lower speed-limit stems from heat production, which in turn depends on power and efficiency. So we have to consider battery voltage, which defines power, and rotational speed, which defines efficiency. The ratio of maximum-efficiency speed and idle speed looks not too complicated:

$$\frac{n_{\eta\,max}}{n_0} = \frac{U_b - \sqrt{U_b \cdot R \cdot I_{0m}}}{U_b - R \cdot I_{0m}} = \frac{1 - \sqrt{\dfrac{R \cdot I_{0m}}{U_b}}}{1 - \dfrac{R \cdot I_{0m}}{U_b}} = \frac{\sqrt{\eta_{m\,max}}}{\dfrac{n_0}{n_{max}}}$$

Interestingly enough, the idle/ideal speed ratio seems to reappear here again twice, now just with a square-root of the proportion of speed lost (by impedance and friction) in the numerator. But the term in the numerator is actually the square root of maximum motor efficiency, which had been implicitly contained in the equation for maximum drive efficiency at the end of the previous section.

That means with a “better” motor (and gear) and a higher battery voltage, peak efficiency and of course the whole efficiency curve are higher. And as we see now, peak-efficiency speed is closer to idle speed.

Generally we can conclude that maximum-efficiency speed is much closer to idle speed than to maximum-power speed (which is always half idle-speed). That means in turn that maximum efficiency is reached at high rotational speed where power is low, so high efficiency and high power are mutually exclusive.

For any given drive, there is quite a difference between the full-power and the cruise-power cases. Since battery voltage is seen as substantially lower in cruise flight, and since it is in the denominator in the equation above, efficiency is lower over the whole rotational-speed range, which is smaller as well. Also, a drive may be used with more or less battery cells and thus voltage, what would make efficiency somewhat higher or lower, respectively.

And as to the difference between “cheap” and “better” drives: For instance the motors’ peak efficiencies may be 0.74 or 0.85, respectively, what looks like quite far from idle speed. But their square roots would be bigger, 0.85 or 0.92, respectively. So both drives have their peak efficiencies close to idle speed, the “better” just even closer, at even less power than a “cheap” drive. Thus, the former has a pronounced peak while the latter’s efficiency curve is rounder and its peak part is flatter.

Then again, a “better” drive is more efficient and produces less heat than a “cheap” one. It may even have more heat-resistant magnets and wire insulation. Hence its tolerable power is higher and its tolerable speed lower, that is closer to maximum-power speed. If full use is made of its power potential, the “better” drive is even further away from its peak efficiency than the “cheap” one. Still its efficiency at tolerable power is better.

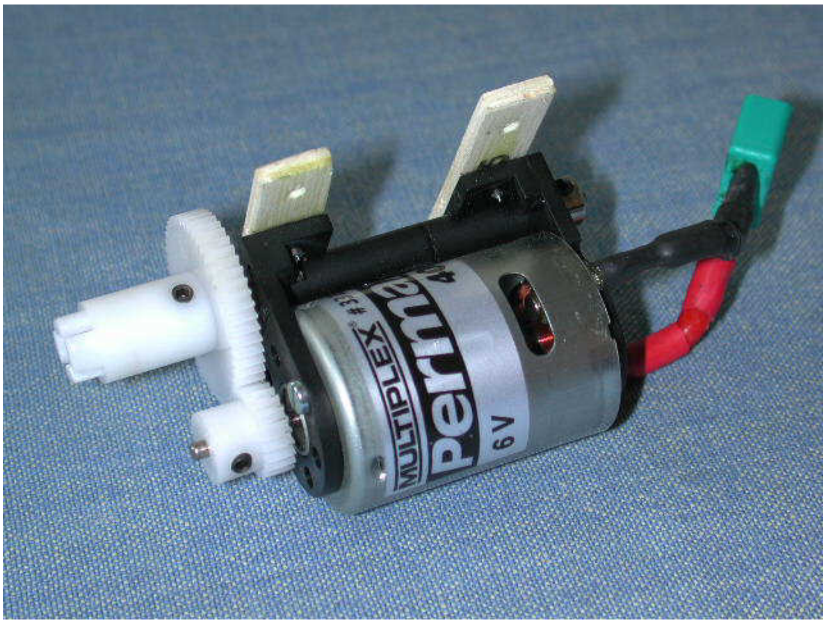
Given that modern motors – brushless, neodymium magnets, ball bearings – are all “better”, and gears as well, this comparison is actually pointless nowadays. There is yet one insight that might be useful: *At full-power setting*, electric model-airplane drives are usually operated quite far from their peak efficiency.

In practical terms, they would reach their peak efficiency only at high dive speeds but never at ordinary flight speeds. The whole drive’s peak efficiency may be typically 5% lower than that of the motor alone, and it is fair to say that drive efficiency in operation is another 10% lower. So motor peak-efficiency is suitable as a *comparative* value for a motor’s quality and it may be used as an advertising point as well, but in any case we have to take it with a grain of salt or just as what it is, respectively.

Motor/Gear Example

Now that all necessary equations are at hand, illustrating diagrams will show all interesting drive characteristics as lines or curves over a rotational-speed (n_g) axis. To this end, a real case has to be chosen as an example, which is as prototypical as possible. In a sense, a drive that is little short of vintage is just that:

It is a drive for a vintage-style parkflyer brought out in 2000. Parkflyers were a new category that made the hobby more affordable and practicable by means of a small and inexpensive electric drive. Characteristic were a 400-size brushed can-motor, a primitive reduction gear for a quite efficient slow-flight propeller, a simple "brushed" ESC, and a 7-cell NiCd battery to be charged from a car battery with a simple charger.



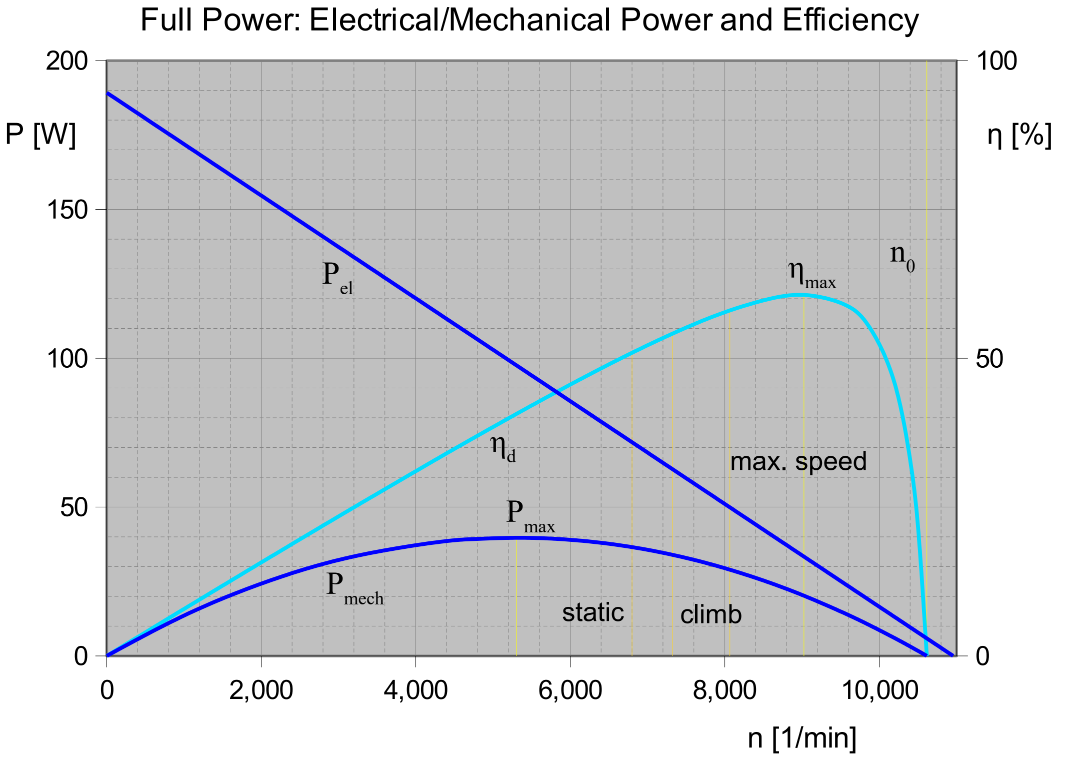
The 7x6.5" propeller was made by Günther, a German manufacturer of flying toys. Actually, this is a toy propeller as well as the gear may be seen as a toy gear. The can motors have been made in huge numbers for automotive applications. All that qualifies the drive as "cheap" in the sense of this chapter.

The calculations described here have been developed for this very drive in the first place. It was not yet customary back then to specify all necessary characteristics. They had to be collected from different sources and derived by own measurements or calculations, respectively. The result in this case is well-nigh typical again:

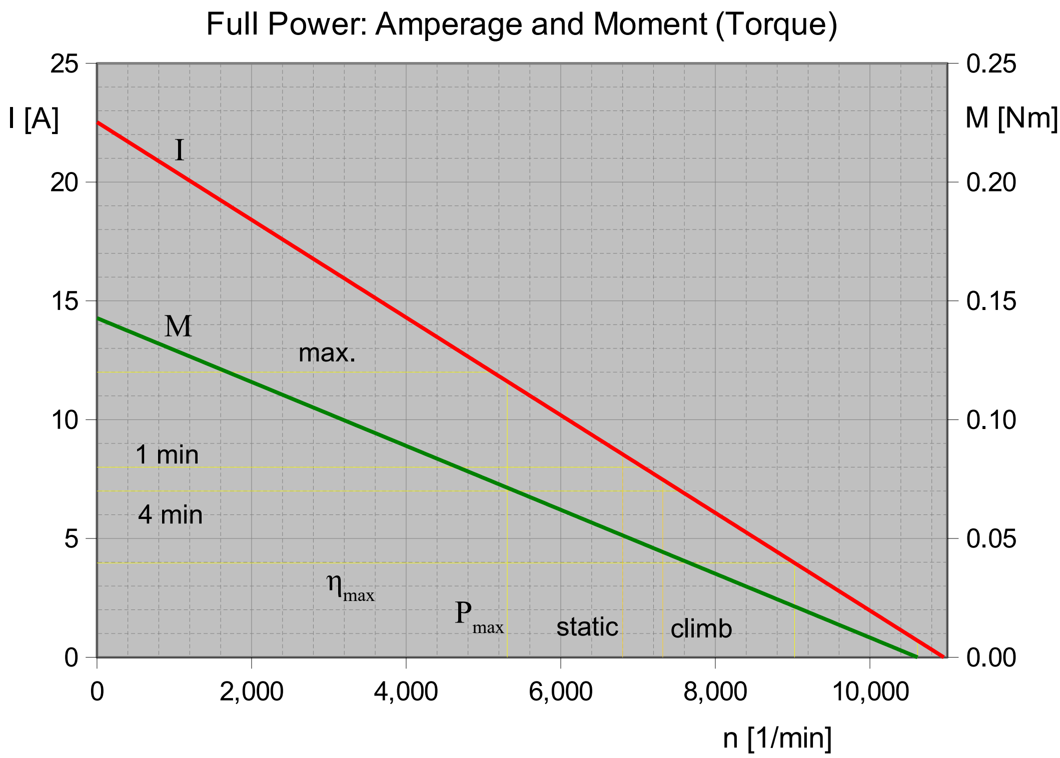
U_b	8.4 [V]	1.2 V nominal NiCd cell voltage, 7 cells
R	0.373 [Ω]	0.24 Ω motor (specified) + 0.133 Ω battery, ESC ("tweaked")
I_{0m}	0.7 [A]	specified, actual value may differ
k_V	3000 [min^{-1}/V]	specified, actual value may differ
i_g	2.3 [-]	specified, actually 49:22=2.227
η_g	0.89 [-]	"tweaked" by experiment and measurement
I_{max}	12/8/7 [A]	absolute/1 minute/4 minutes, loosely specified

Motor/Gear Diagrams

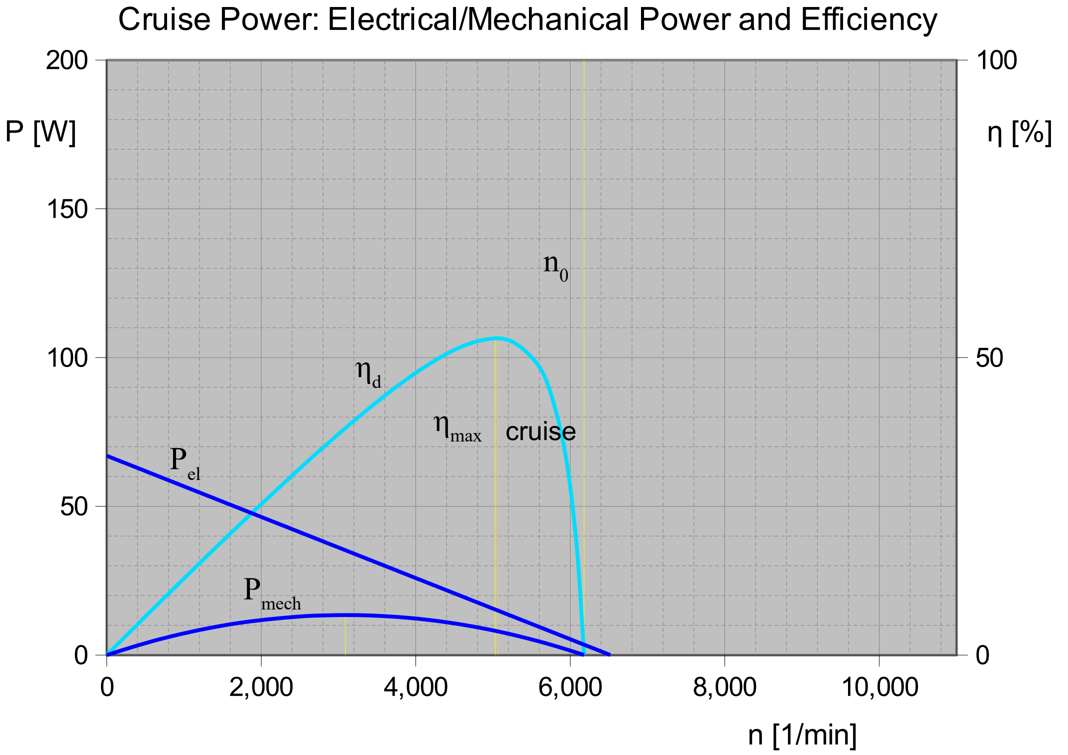
While its peak value is even 61%, overall drive efficiency is only 58% at maximum straight-and-level speed and 54% in climb (in this case of a retro-style parkflyer):



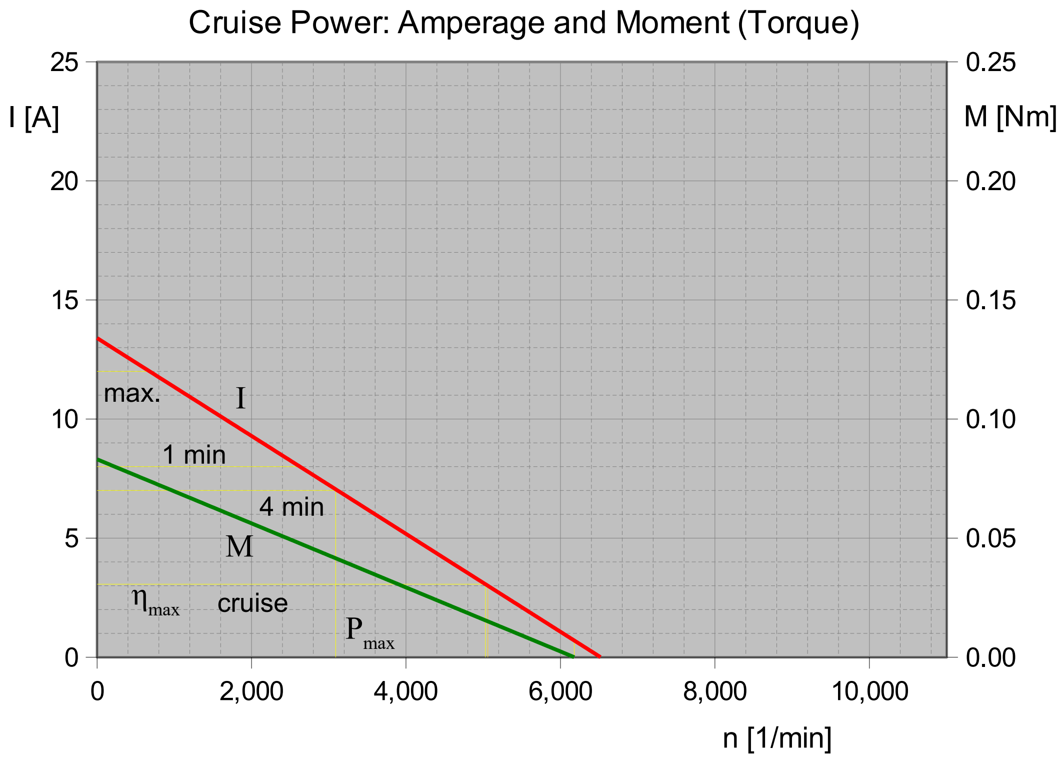
Coincidentally (in this case), maximum tolerable amperage is even at a slightly slower rotational speed than maximum power, but static run is slightly beyond the 1-minute amperage limit, and climb is slightly beyond the 4-minute amperage limit:



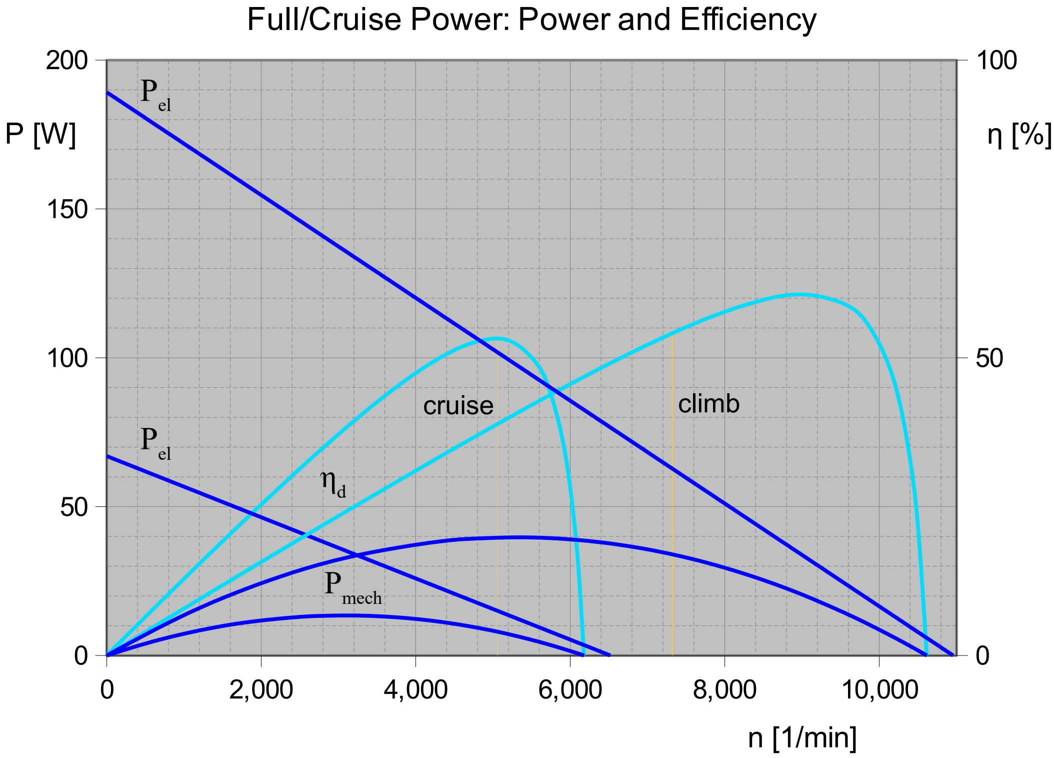
The diagrams on the previous page show the case of full-power, that is 8.4 V battery voltage. The following two diagrams show the case of cruise power (which is known from the performance calculations as well as the climb case). The ESC is set for an equivalent 0.6 voltage reduction factor, giving 5.0 V equivalent battery voltage:



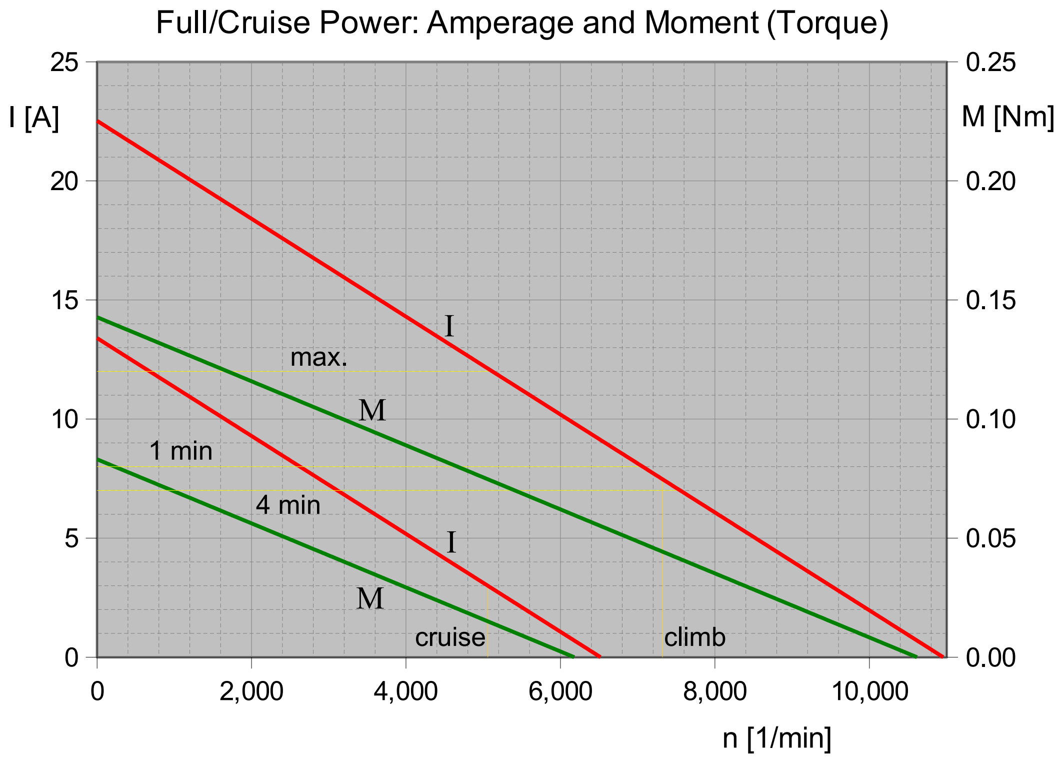
Cruise rotational speed advantageously coincides with maximum-efficiency rotational speed. That may be just a coincidence, but it might have been deliberate designing as well. The amperage limits still all apply but are not relevant in cruise flight:



Finally, we compare the full-power and cruise-power cases. The lines of electrical power are not parallel (just an observation), and the efficiency curves (particularly peak efficiencies) are different, both due to the different voltages. However, the efficiencies in cruise and climb are virtually equal (53% or 54%, respectively):

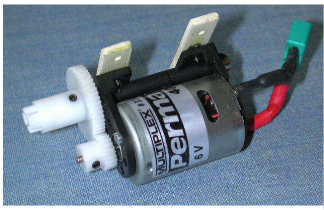


The lines of amperage and moment (torque), respectively, *are* parallel. That means the drive's elasticity (its decrease of rotational speed with increase of load) does not depend on power setting. Cruise and climb currents (3.0 A, 7.5 A) are worth noting:



Motor/Gear Comparison

There are no hard and fast rules about drawing conclusions from drive characteristics, yet there are striking similarities in different cases. To give a clue, the vintage "cheap" example drive is compared to two rather different ones. The first comparative example is vintage as well, just a "better" brushed inrunner drive. The second is ten years younger and nowadays typical with brushless/gearless outrunner motor and a LiPo battery. Efficiencies and a few ratios are worth noting:



Type of model	55" retro parkflyer	100" thermal glider	95" Sr. Telemaster
Weight of model	0.85kg / 1.9lbs	1.7kg / 3.75lbs	4.5kg / 10lbs
Motor	400-size "can"	480-size premium	4130 brushless
Gear i_g – η_g	2.3:1 – 89%	4.4:1 – 95%	no gear – 100%
Weight of drive	95g / 3.35oz	184g / 6.5 oz	405g / 14.3oz
Power "in" static	70 W (82 W/kg)	150 W (88 W/kg)	500 W (111 W/kg)
Power "out" static	37 W (0.39 W/g)	100 W (0.54 W/g)	350 W (0.86 W/g)
Battery (weight)	7s 1000 NiCd (170g)	7s 2300 NiCd (442g)	4s 5000 LiPo (548g)
B. Voltage (energy)	8.4 V (49 Wh/kg)	8.4 V (44 Wh/kg)	14.8 V (135 Wh/kg)
Motor / Drive k_V	3000 / 1300 rpm/V	3440 / 780 rpm/V	360 / 360 rpm/V
Idle/max. I_{0m} / I_{max}	0.7 A / 8 A 1min	0.76 A / 20 A 1 min	1.3 A / 60 A 1 min
Motor / total R	0.24 / 0.373 Ω	0.071 / 0.134 Ω	0.062 / 0.117 Ω
Peak eff. $\eta_{d\,max}$ ($\eta_{m\,max}$)	61% (74%)	75% (85%)	81% (86%)
Cruise/climb eff.	53% / 54%	66% / 65%	73% / 71%
Cruise/climb amps	3.0A / 7.5A = 0.40	3.7A / 16.9A = 0.22	9.2A / 33.6A = 0.27
Cruise/climb rpm	5050 / 7250 = 0.70	2400 / 4800 = 0.50	2150 / 3900 = 0.55
Climb/ideal rpm	7250/10920 = 0.66	4800 / 6550 = 0.73	3900 / 5330 = 0.73

The first drive is so weak and inefficient that its amperage in cruise has to be even 40% of that in climb. Now cruise rpm is even 70% of climb rpm, and climb rpm is only 66% of ideal rpm. In more "normal" cases like the two other drives, about ¼, ½, and ¾, respectively, would be good first-order estimates for these ratios.

The "better" the drive the better are all its efficiencies and the lesser is the difference between motor and drive peak-efficiency. The respective efficiencies in cruise and climb are about equal, and up to ten percent-steps lower than peak efficiency. There are size effects, but they are small: A brushless/gearless replacement for the small first drive has two percent-steps less peak efficiency than the big third drive.

Digression: Helicopter

Finally we take a look at an example that does not actually fit in our comparison since it is a helicopter motor, as indicated by the pinion on its shaft. Yet it is an interesting example for a motor's friction moment and it would make a good tow-plane motor.

It is very powerful for its size and weight by means of special magnetic material, thin-sheet laminated core, thick non-stranded wire, heat-resistant insulation and magnets, cooling fan, and high rotational speed. This is not only a "better" motor, this is a "premium" one, evidenced by efficiency and price. A few characteristics were specified, but idle current and efficiency were not:

Power "in"	2000 W continuous (max. with 8s LiPo)
Speed	30000 rpm maximum (mechanically)
Weight	235g / 8.3oz
Battery	6s LiPo / 22.2 V (or 8s / 29.6 V)
spec. speed k_V	930 rpm/V
Resistance R_m	0.011 Ω
Idle amps I_{0m}	0.8 A at 500 rpm 2.7 A at 15000 rpm
Peak eff. $\eta_{m\,max}$	93% at 20000 rpm (6s) 94% at 27000 rpm (8s)

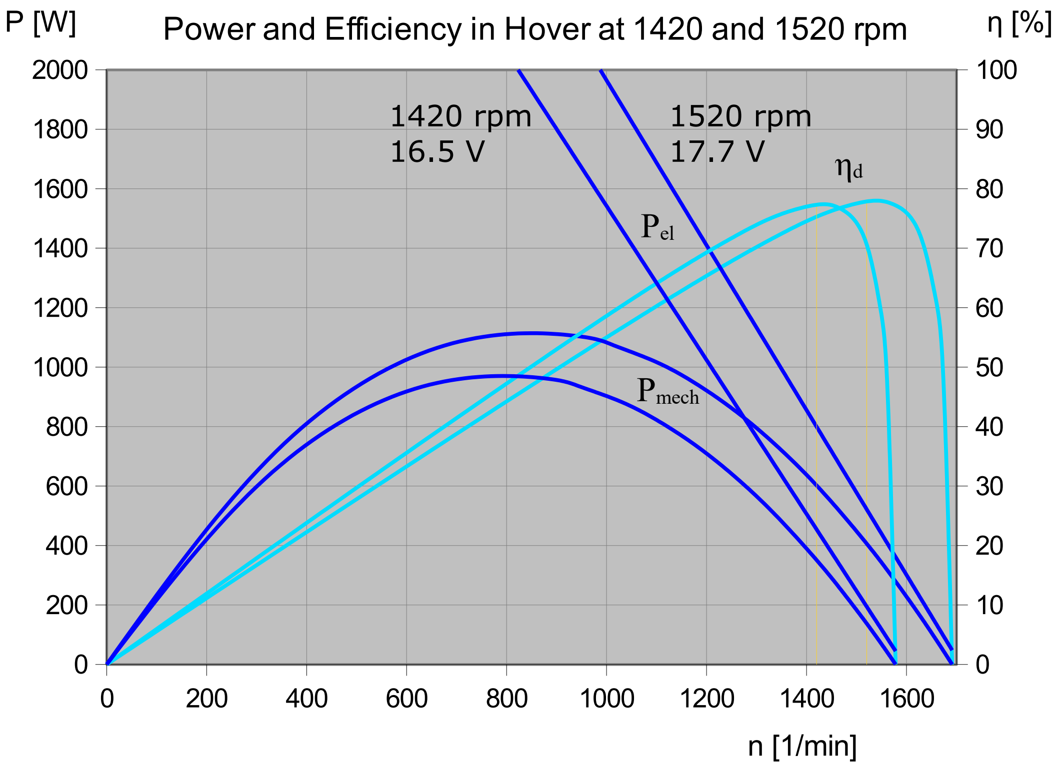
Idle current was measured by spinning the unloaded motor with the dedicated battery and ESC, which is actually a governor and which has built-in telemetry. I_{0m} turned out to be highly variable, unlike in our simplified drive model where – for a linear model – it is just assumed to be constant *in the operational speed range*, which is small. (That is shown in the example full-power diagram above, although the substantially lower cruise-power speed might make a different I_{0m} value advisable.)

Only two values were taken, 0.8 A at a speed close to zero (500 rpm) and 2.7 A at target speed (15000 rpm). Using the latter as the constant value in the simplified drive model results in a still spectacular 93% motor peak efficiency at 20000 rpm (and even 94% at 27000 rpm with an 8s LiPo battery). The lower value (0.8 A at 500 rpm) just goes to show how variable idle current I_{0m} actually is.

The helicopter's transmission consists of a 9.625:1 reduction-ratio main-rotor gear, a subsequent 1:5.5 transmission-ratio tail-rotor gear, a gimbal joint, and a 1:1 ratio angular gear. The whole transmission's efficiency is higher than 96%. A single-stage or even a two-stage tow-plane gear, built like a helicopter's main-rotor gear or as a special planetary gear, would be at least as efficient (without the tail-rotor gear).

While an airplane is borne by its wings and driven by a propeller, a helicopter is borne as well as driven by its main rotor, which is a rotary wing. Other than a model propeller, it has variable blade-pitch, both cyclic and collective, used to control the helicopter's pitch and bank as well as acceleration or climb (and the respective opposites). Although the "thrust" needed from the main rotor is highly variable, its average approximates the helicopter's weight.

The main rotor can produce the same thrust at low rotational speed and high blade pitch or – vice versa – at high rotational speed and low blade pitch. For reasons to be explained only in the next chapter, the main rotor needs less power at lower rotational speed. In our simplified drive model, the ESC adjusts speed by the equivalent of more or less battery voltage. Virtually proportional to any voltage change, all curves in the motor/gear diagram are scaled horizontally and the power curves vertically as well. Efficiency is only marginally affected (vertically) by voltage change:



The three-blade, 1092 mm diameter main rotor produces the thrust for 4.5 kg weight either at 1420 rpm rotational speed with 450 W electrical power or at 1520 rpm with 520 W. Overall efficiency η_d is 77% or 78%, respectively, the pronounced maximum in both cases. This example shows that helicopter drives are always operated (on average) at the same points on the curves in the diagram, no matter how these curves are scaled. And it shows that this motor (size, k_V) is chosen to work at its peak efficiency.

To this end, it has to spin relatively fast, that is close to the respective maximum, not exploiting either its power potential (only 500 W of up to 2000 W are used on average) or its speed potential (only 14500 rpm of up to 30000 rpm). Together with a high-ratio gear and a proper high-pitch propeller in a low-wing-loading tow-plane, this motor should further exploit its speed potential (by higher voltage, 8s instead of 6s) though still not its power potential (in order to work at peak efficiency). That would make for about 80% efficiency in climb. (Compare examples in previous section.)

This is possible only because the motor is very powerful and efficient but still very small and lightweight, even if expensive – just a modern "premium" type.

Propeller Illustration

Propeller Data

We have to employ a specialized software tool to calculate a propeller's coefficients, or these are offered by the propeller's manufacturer, or there are even coefficients measured in a wind tunnel. In any case, we get a table with values for at least the advance ratio J, the power coefficient c_P, and the thrust coefficient c_T. There may be more values that are useful to assess the propeller, but they are not used in the drive calculations. For our example propeller, JavaProp calculated the following data:

J	c_P	c_T	η	η*	stalled
[-]	[-]	[-]	[%]	[%]	[%]
0.00	0.12445	0.13799	0.0	0.0	100 !
0.05	0.08813	0.12009	6.8	16.3	100 !
0.10	0.10927	0.14835	13.6	27.1	100 !
0.15	0.11751	0.15737	20.1	36.7	100 !
0.20	0.12102	0.15838	26.2	45.3	100 !
0.25	0.12248	0.15489	31.6	52.9	94 !
0.30	0.12170	0.14826	36.5	59.8	66
0.35	0.11894	0.14045	41.3	65.9	38
0.40	0.10057	0.12027	47.8	72.7	5
0.45	0.09208	0.10832	52.9	77.7	0
0.50	0.08585	0.09705	56.5	81.9	0
0.55	0.07852	0.08472	59.3	85.7	5
0.60	0.07029	0.07188	61.4	88.9	5
0.65	0.06112	0.05839	62.1	91.8	11
0.70	0.05088	0.04423	60.9	94.3	11
0.71	0.04867	0.04128	60.2	94.8	11
0.72	0.04643	0.03830	59.4	95.2	11
0.73	0.04420	0.03524	58.2	95.7	16
0.74	0.04189	0.03225	57.0	96.1	16
0.75	0.03955	0.02924	55.5	96.5	16
0.76	0.03711	0.02614	53.5	96.9	16
0.77	0.03462	0.02300	51.2	97.4	16
0.78	0.03212	0.01988	48.3	97.8	16
0.79	0.02959	0.01676	44.7	98.1	16
0.80	0.02709	0.01346	39.8	98.5	22
0.81	0.02441	0.01021	33.9	98.9	22
0.82	0.02167	0.00692	26.2	99.3	22
0.83	0.01894	0.00366	16.0	99.6	22
0.84	0.01616	0.00038	1.9	100.0	22
0.85	0.01327	-0.00302	-19.3	100.0	22

The coefficients depend on the advance ratio. From zero, that is zero speed (static), to the first value at which the propeller delivers negative thrust, the advance ratio is incremented by a variable step width. So the number of steps, or rows in the table, depends on the propeller and the software tool. Wind tunnel measurements may cover only part of the whole range of advance ratios and the static case (zero).

The table above is an example for a certain propeller as well as for a certain calculation tool. It is only an excerpt as far as all absolute values (rpm, speed, power, thrust) have been omitted. In the following discussion of the table's columns, literal quotations from the JavaProp users manual are enclosed in » and « quotation marks:

»The propeller analysis is performed at fixed intervals of J but the step size is adapted and reduced when the efficiency begins to drop.«

The power coefficient c_P and the thrust coefficient c_T for each advance ratio J are the values actually needed for the drive calculations.

Propeller efficiency η (named η_p in the other chapters) is calculated from the first three columns using the simple equation specified in the section Mechanical-Aerodynamic Conversion. »The output also contains values of η* which is the maximum possible efficiency for the current power loading.« This is called propulsion efficiency, reflecting only the power lost in the propeller's slipstream (or wake) by repulsion.

»One column is labeled "stalled" – it lists the percentage of the blade where the local airfoils are operating at angles of attack beyond stall. An additional exclamation mark "!" appears in this column when the power loading is too high for the theory to give accurate results. This usually happens at low advance ratios.«

See Martin Hepperle's JavaProp Users Guide.

Usually, propeller coefficients are calculated or measured, respectively, for various advance ratios J *at a fixed rotational speed* n. To this end, flight speed v is varied from zero to the speed where thrust is zero (colloquially called "pitch speed"). Low flight speeds or advance ratios, respectively, mean high angle-of-attack on the blades and a high power loading from lift and drag, especially when the blades are stalled.

No propeller analysis tool is able to calculate reliable coefficient values in the realm of stall. That is why JavaProp warns with an exclamation mark "!" when the better part of the blades is stalled at low speed. That does not mean that the calculated coefficients are completely useless but that they are far from accurate. Actually, the coefficients are not quite reliable even at higher speeds as long as there is at least some stall. In this example, only the 0.45 and 0.50 advance ratios are without any stall.

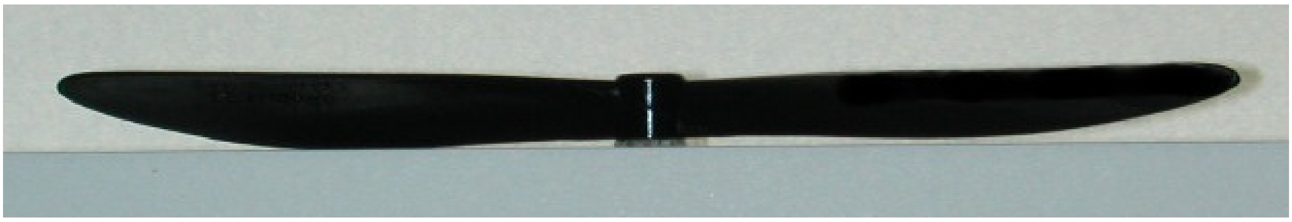
At even higher speeds or advance ratios, respectively, an ever increasing inner part of the blades is (negatively) stalled. That does not really spoil the coefficients because the inner part contributes little to the propeller's thrust and torque. At low advance ratios, the whole blades are more or less (positively) stalled, particularly the outer parts which make for most of thrust and torque. As shown in the next section, the blade's pitch increases from hub to tip so only a small range of advance ratios gives a good angle-of-attack on the whole blade.

At higher *rotational* speed, the blades are basically working in a faster, hence "better" airflow and can generate more thrust. As a result, thrust coefficients are higher in a wider range of advance ratios. However, the difference is relatively small and an electric drive has little increase of rotational speed n (colloquially called "unload") at higher flight speed v. So, coefficients calculated or measured for only one rotational speed will suffice for the whole flight speed range in drive calculations. This fixed rotational speed should just be close to the actual rotational speeds occurring on the drive in flight. It is even acceptable to use coefficients provided for full-power rotational speed for the much lower cruise-power speed as well since calculated coefficients hardly differ (other than measured ones).

Propeller Example

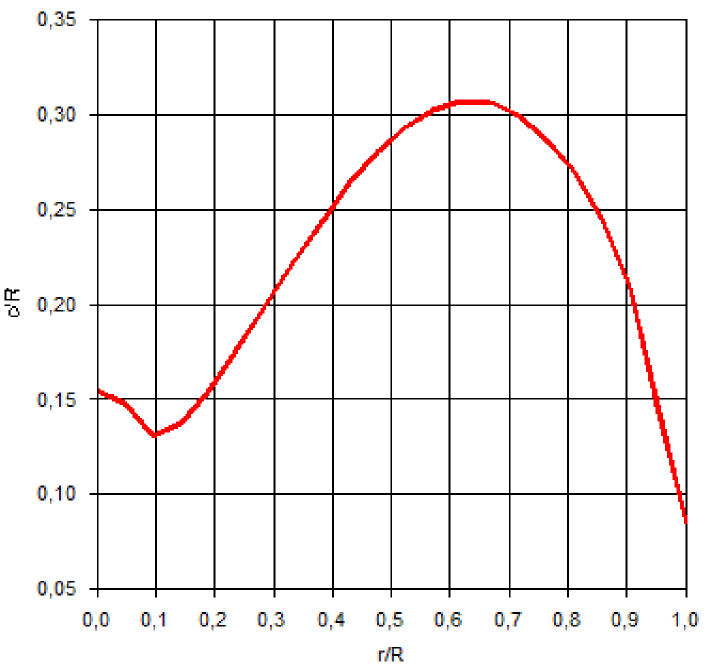
This 7x6.5" propeller (actually 17.5x16 cm) has been mentioned and shown before in the Motor/Gear Example section. It was made by Günther, a German manufacturer of flying toys. Together with the primitive gear, it was optimized for 400-size parkflyer drives and has an unusually high pitch-to-diameter ratio. The blades have a traditional elliptic planform and a flat-plate airfoil with a defined leading edge radius. This simple (as opposed to refined) design makes its characteristics well predictable for a simplified calculation tool, even if still not in cases where blade stall occurs.

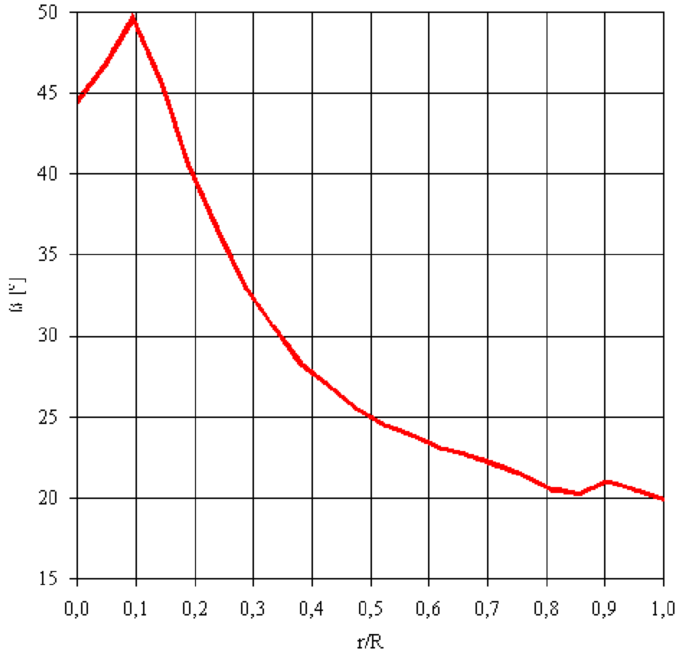
A front view and a side view have been shot with a telephoto lens to minimize perspective distortions. Lens distortions have been removed with a special software tool. The black propeller makes for good contrast but still the outlines had to be improved in a graphics editor. Both propeller pictures must have the same width (in pixels) so they can be processed by Martin Hepperle's PropellerScanner. This program combines both blades into an abstract blade geometry in table form.



Additionally there are diagrams for local blade chord c, twist angle β, and pitch H. They are all drawn over the blade's relative radius r/R from center (0.0) to tip (1.0) and are useful to check the geometry derived from the pictures for correctness or plausibility, respectively.

This diagram shows blade chord derived from both front and side view. Outline errors are small compared to the chord seen in the top view so the result is a smooth line showing the hub and the blade's round shape.

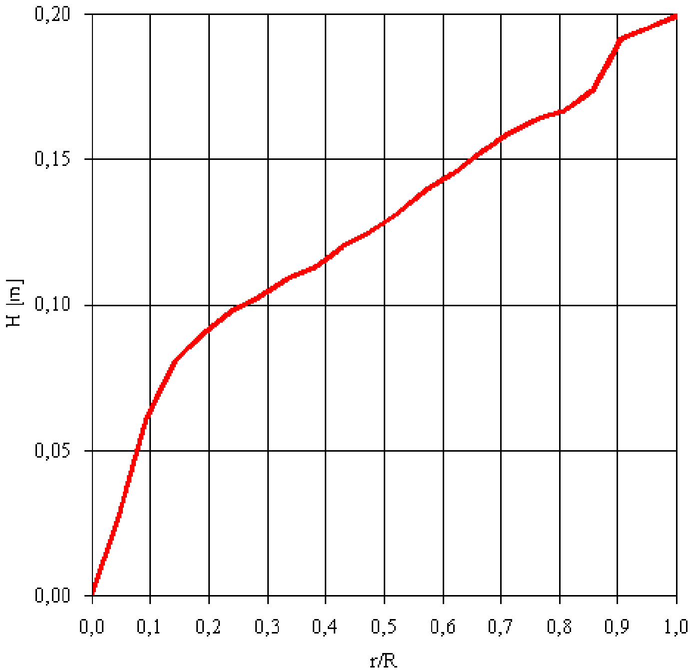




The twist angle diagram shows the hub with a defined angle, what is obviously wrong. It has to be clipped later in the calculation tool by specifying the hub's diameter.

Expectably, blade angle β should be close to 90° at the center and should diminish towards the tip so that it corresponds to the propeller's pitch H.

Essentially, β is the arc tangent of the pitch-to-radius ratio. This in turn is a hyperbolic line if local pitch H is assumed constant and local radius r/R runs from 0.0 to 1.0 or r from 0 to R, respectively. This highly curved hyperbola combined with the arc tangent gives the characteristic curve shape shown here.



Usually the curve is more or less skewed because pitch H is actually not constant over the radius r/R. For different reasons, propeller designers choose varying local pitch values H from hub to tip.

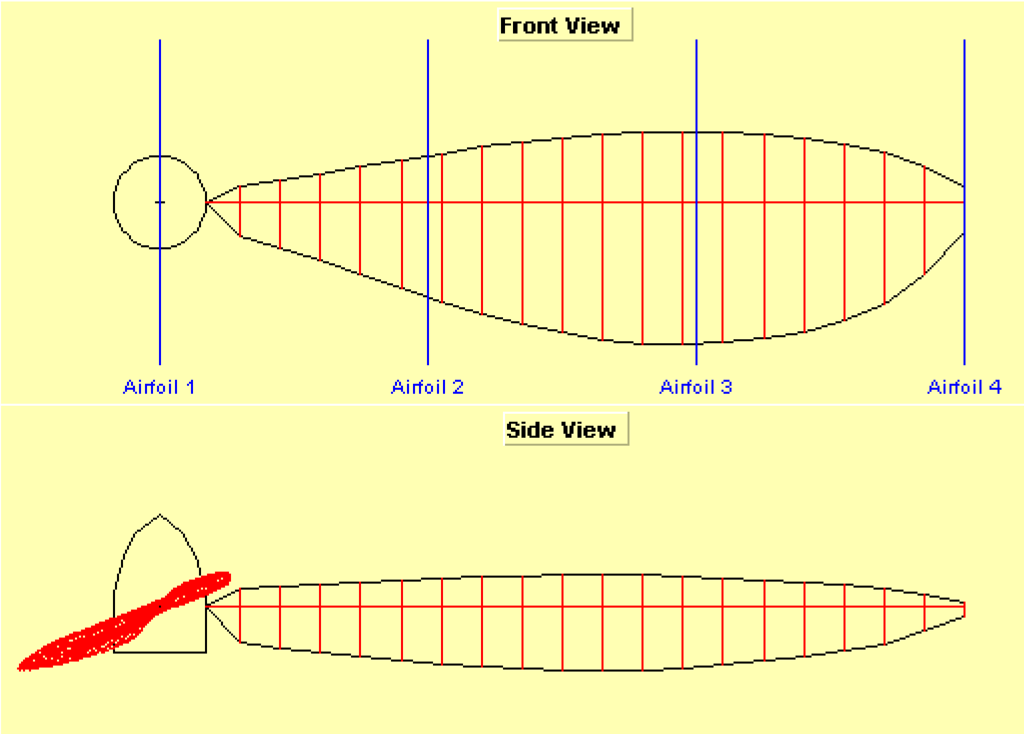
In this case, local pitch H increases linearly from hub to tip as shown in this diagram. The 0.16 m nominal pitch occurs at 70% of the blade radius R while rather 75% of radius is the common reference point for propeller designs.

Above $0.70 \cdot R$, towards the tip, the local pitch is bigger than the nominal one, and below this point, towards the hub, it is smaller. Accordingly, the local twist angles are bigger or smaller, respectively.

In both diagrams, the lines are noticeably uneven between about 0.7 and 1.0 relative radius r/R. Improving the outlines in a graphics editor went not really well especially on the right blade in the side view. The outline drawn there by hand turned out uneven, but only half of the error is in the pitch and twist values because they are derived from both blades.

The unevenness could have been corrected in the pictures or in the geometric data derived from them. This has not been done because the error is small, particularly since the whole propeller calculation is only an approximation. Among other things, this example is supposed to show the relative insignificance of such errors.

The geometry table produced by PropellerScanner has to be reduced to three columns with radius r, blade chord c, and twist angle β. This excerpt can be entered into Martin Hepperle's JavaProp calculation tool. At least the propeller and spinner diameters have to be specified as well as a rotational speed for the propeller analysis. A sketch shows the propeller's geometry as used for the calculations:



The hub is drawn like a spinner and except from the calculations. Front, top, and side view show blade elements aligned to the same proportion of their chord (33% by default). In case of this example propeller, that matches the real shape quite well. Scimitar-shape propellers would need alignment to up to 100% or even more. Anyway, the sketch shows smooth outlines and twist angles with little unevenness.

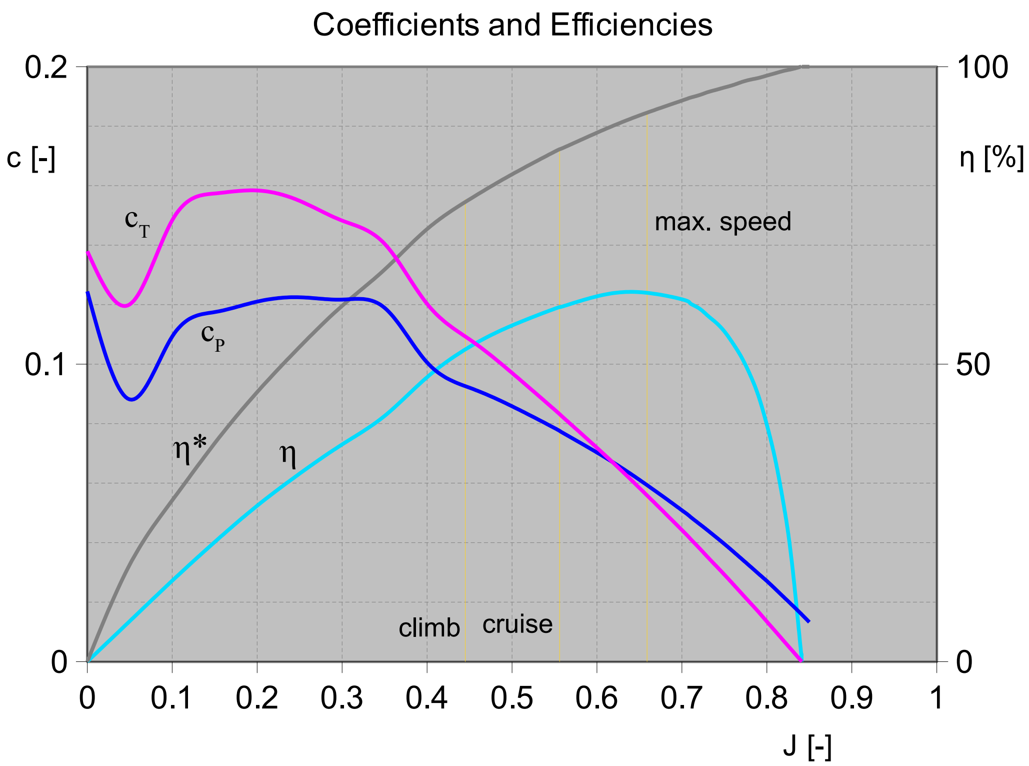
Airfoils have to be specified for the four radius stations shown in the sketch (at center, one-third and two-thirds of radius, and tip). There are lift and drag coefficients for several airfoils prepared in the tool, and it is possible to add some for more. That is not necessary here since the example propeller has flat-plate airfoil over the whole radius. There are even two sets of flat-plate coefficients – for Reynolds numbers 100,000 and 500,000 – but there is only a small difference in drag. Besides, on this small and slowly spinning propeller the Reynolds number goes up to only 80,000 (shown by PropellerScanner) so 100,000 at all four stations is the single choice.

There are several inaccuracies now: (1) The outer blade parts are rendered somewhat uneven, though the general outline is correct and even fits the pitch specification. (2) Airfoil coefficients are available only for too high Reynolds numbers, though flat-plate is fairly insensitive to them so the values used should be not too optimistic. (3) Rotational speeds are different in climb and cruise (7300 / 5100 rpm), though the difference in propeller coefficients should be only a few percent.

Comparative calculations showed that indeed the *calculated* propeller coefficients vary very little. We conclude that propeller calculations are in no case exact, not even in this simple case. Then again, probably nobody would measure this toy propeller in a wind tunnel, so calculation is the only choice. In view of its limited accuracy already, the results for 7000 rpm are used for any rotational speed in question here.

Propeller Diagram

The example propeller's coefficients had to be calculated. The analysis for 7000 rpm rotational speed yielded the data listed in the first section of this chapter, which are shown (except the last column) in the following standard propeller diagram:



The two coefficients c_P and c_T as well as the two efficiencies η and η* are plotted over the propeller's whole range of advance ratios J from zero ("static") to 0.84 ("pitch speed"). Rather, this range should reach at least to the 0.91 pitch-to-diameter ratio, which is unusually high for a model propeller and actually advantageous here.

The problem is that peak efficiency being at only 0.65 advance ratio limits ideal (theoretical) peak efficiency to only 75%. (That is just a statement here, without explanation.) So this "cheap" propeller gives away the potential for better efficiency, presumably due to its particular local-pitch distribution and flat-plate airfoil. A most efficient propeller would have 1.4 maximum advance ratio, its peak efficiency being at 1.0 advance ratio. There, ideal (theoretical) peak efficiency is 83%, the absolute (physical) maximum even for a "perfect" propeller (another mere statement).

The lines for the coefficients c_P and c_T look smooth above 0.45 advance ratio but warped below. That is where serious blade stall occurs so calculated coefficients are unreliable. The lines now clearly show *how* unreliable that might be. Advantageously, the operating points for climb and cruise (known from the performance calculations) are in the smooth, reliable range.

That is typical for fairly well designed propellers, as well as the basic shape of the lines. Very roughly, they have a horizontal part where blade stall occurs and go down where not. Propellers without any blade stall in the higher advance-ratio range (unlike this one which has some above 0.50) actually have a virtually straight c_T line there. Of course, when thrust (c_T) is zero (at "pitch speed") some positive power (c_P) is still needed to overcome the blades' drag.

The efficiency η line is only faintly warped because the warps in the c_P and c_T lines are very similar. These coefficients represent something like lift and drag on the propeller blades, respectively, so calculation errors in the realm of stall affect both similarly. Hence, dividing one by the other makes for a nearly smooth efficiency curve.

Its shape shown here is typical. Efficiency, as the ratio of thrust power and shaft power, is zero at zero flight speed because – despite big thrust – thrust power is zero there. Conversely, thrust is zero at "pitch speed" and so are thrust power and efficiency again.

The curve is skewed to the right with the peak efficiency at a relatively high advance ratio, just like an electric drive's efficiency curve (see previous chapter) and for analogous reasons. Correspondingly, in a full-power *climb* the propeller is highly loaded and does not work at its best efficiency. The propeller operates quite lightly loaded and close to its peak efficiency in *cruise* flight.

The efficiency η^* line is only faintly warped as well (for the same reasons as η). It is a measure of the power dissipated in the propeller's slipstream when generating thrust by repulsion (Newton's second law). Hence it is zero at zero flight speed because all the power put into the slipstream makes for thrust but not for thrust-power. It goes up to 100% when thrust is zero at "pitch speed" because there is no slipstream then.

A lightly loaded propeller (low *power* per propeller disk area) accelerates its slipstream only slightly. That is more efficient than producing the same thrust by strongly accelerating a smaller, hence higher-loaded propeller's slipstream. At 7000 rpm, this example propeller is highly loaded with c_P values higher than 0.10 at advance ratios lower than 0.5. Its η^* curve is not far from a straight, diagonal line between its end points. A lightly loaded propeller would have c_P values all below 0.10, an η^* curve more curved away from a diagonal, and a correspondingly more "bulgy" η curve.

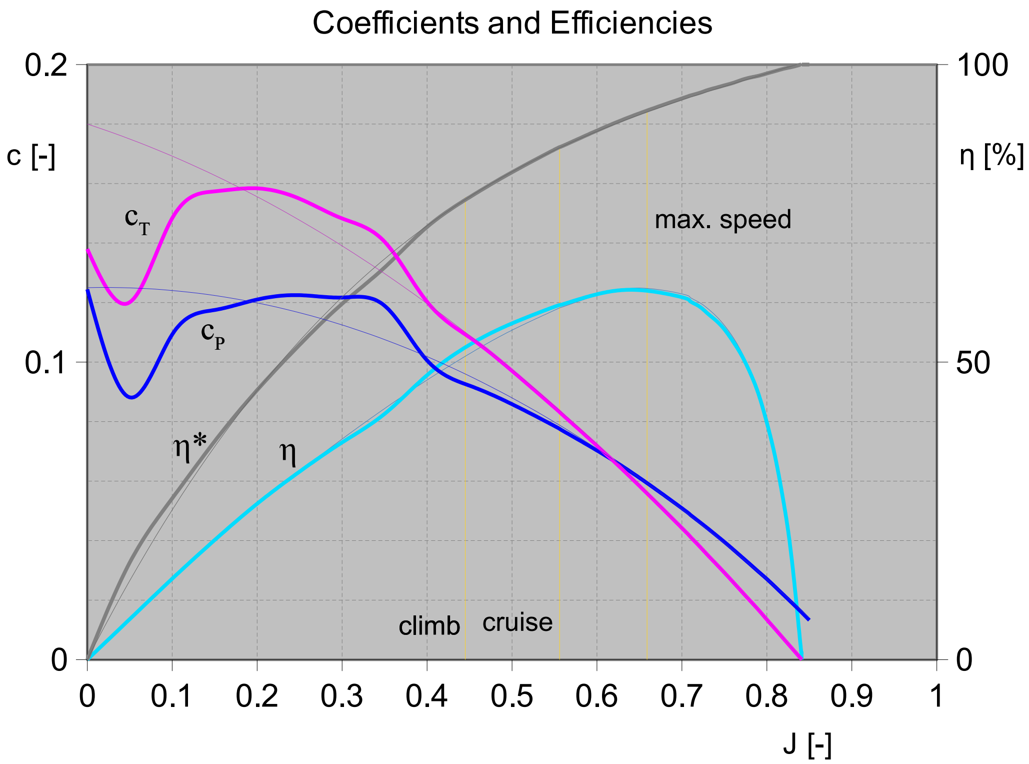
In the dimensionless propeller diagram, the η^* line shows an "ideal" efficiency, that is with repulsion losses only. Thus, the closer the η line is to the η^* line, the better is the propeller's design. Besides, the lighter loaded a propeller is, the more curved are both lines and good efficiencies are possible over a wider range of advance ratios. And finally, the higher the advance ratios the propeller can reach, the higher the efficiency curve can go. A perfect propeller's peak efficiency would be 83% at 1.0 advance ratio. Customary propellers, even "better" ones, reach their noticeably lower peak efficiencies at advance ratios lower than 1 (see comparisons in the next sections).

This example propeller is far from perfect, with its elliptic blade shape, round leading edge, and flat-plate airfoil. It is highly loaded because it is small and spins fast, so it has moderate peak efficiency in a quite small speed range. Its peak efficiency is moderate also because it reaches only low advance ratios due to its particular local-pitch distribution and its flat (not cambered) airfoil. It is just a typical "cheap" propeller.

Still the designer managed to get the most out of it. Both motor and propeller operate close to peak efficiency in cruise flight. Efficiency in climb is still acceptable and climb is only a minor part of the airplane's flight envelope. So the propeller as well as the whole drive (and the airplane) are not *technically* ideal but *economically*. This is the kind of insight to expect from these drive calculations. After all the drive is not designed or optimized here but just *analyzed*.

Now that we know the example propeller is not technically ideal, there is actually no reason to strive for ideal calculations. In addition to the inaccuracies mentioned in the previous section, there are the coefficient-curve warps in the lower advance-ratio range. It is technically possible and hence tempting to smoothen these curves, so we must finally show that this would be to no avail.

To this end, the example standard propeller diagram is repeated with smoothened curves added. For simplicity's sake, the spreadsheet software's built-in polynomial interpolation function has been used. Simple second-order (quadratic) polynomials give a very good approximation in the higher advance-ratio range and a reasonable, smooth curve in the lower. The automatic polynomial-coefficient calculation needed some help in the form of preset axis intercepts, which have been found by trial. In the diagram, the smoothened curves are thinner and darker than the original ones.



The smoothened efficiency η curve is an exception in that it is not interpolated. Its values have been calculated by employing the usual equation, that is dividing the smoothened c_T values by the smoothened c_P values and multiplying by the advance ratio J. Hence the efficiency curve is visual proof of the smoothened coefficient curves' plausibility.

Yet these curves are of little value. Since the coefficient ratio is multiplied by the advance ratio, even aberrant coefficient values give reasonable efficiency values, the more so the lower the advance ratio is. But warps are just in the low advance-ratio range while at higher advance ratios the coefficient curves are smooth, anyway.

The smoothened c_P curve's axis intercept happens to coincide with that of the original curve. Then again, the smoothened c_T curve's axis intercept is substantially higher than the static thrust coefficient calculated by JavaProp. Even if this calculation is unreliable due to the occurrence of blade stall, it is actually not wrong. Rather it seems that the smoothened curve would pertain to an *ideal* propeller with little to no blade stall. In fact, the *real* propeller's static thrust has been measured with a spring scale and it turned out to be approximately like calculated.

In the end, there is just no way to get correct coefficient curves. The warped curves are still useful after all, the more so since their relevant part is smooth, anyway. And if reliable static values are needed they can be measured.

Propeller Comparison

This section corresponds to the respective comparison section in the previous chapter. The example propeller, a Günther 17.5x16cm toy propeller, belongs to the example drive. It is compared to the respective propellers of the other two drives, an aero-naut CAM-Carbon 14x8" folding propeller and an APC 17x12" E thin electric propeller. They are shown to their relative sizes here:

Type of model	55" retro parkflyer	100" thermal glider	95" Sr. Telemaster
Weight of model	0.85 kg / 1.9 lbs	1.7 kg / 3.75 lbs	4.5 kg / 10 lbs
Diam. x pitch – ratio	6.9x6.3" – 0.91	14x8" – 0.57	17x12" – 0.71
Disk area – max. η_p	0.024 m^2 – 62%	0.099 m^2 – 75%	0.146 m^2 – 69%
Opt./max. J – ratio	0.65 / 0.84 – 0.77	0.55 / 0.70 – 0.79	0.80 / 1.09 – 0.73
climb/cruise Speed	9.6 / 8.0 m/s	11.7 / 9.0 m/s	15.0 / 12.0 m/s
Thrust	1.86 / 0.65 N	5.46 / 0.74 N	14.3 / 2.76 N
Thrust power	17.9 / 5.2 W	63.8 / 6.6 W	214 / 33 W
Rotational speed	7336 / 4931 rpm	4836 / 2651 rpm	3911 / 2171 rpm
Advance ratio	0.45 / 0.56	0.41 / 0.57	0.53 / 0.77
Thrust coefficient	0.10832 / 0.08329	0.04322 / 0.01961	0.07896 / 0.04943
Power coefficient	0.09208 / 0.07761	0.02567 / 0.01511	0.06878 / 0.05445
Moment (torque)	4.4 / 1.7 N·cm	18.3 / 3.2 N·cm	85.8 / 20.9 N·cm
Shaft power	34 / 9 W	93 / 9 W	351 / 48 W
Power loading	1409 / 360 W/m^2	935 / 90 W/m^2	2397 / 324 W/m^2
Efficiency	53% / 60%	69% / 74%	61% / 69%

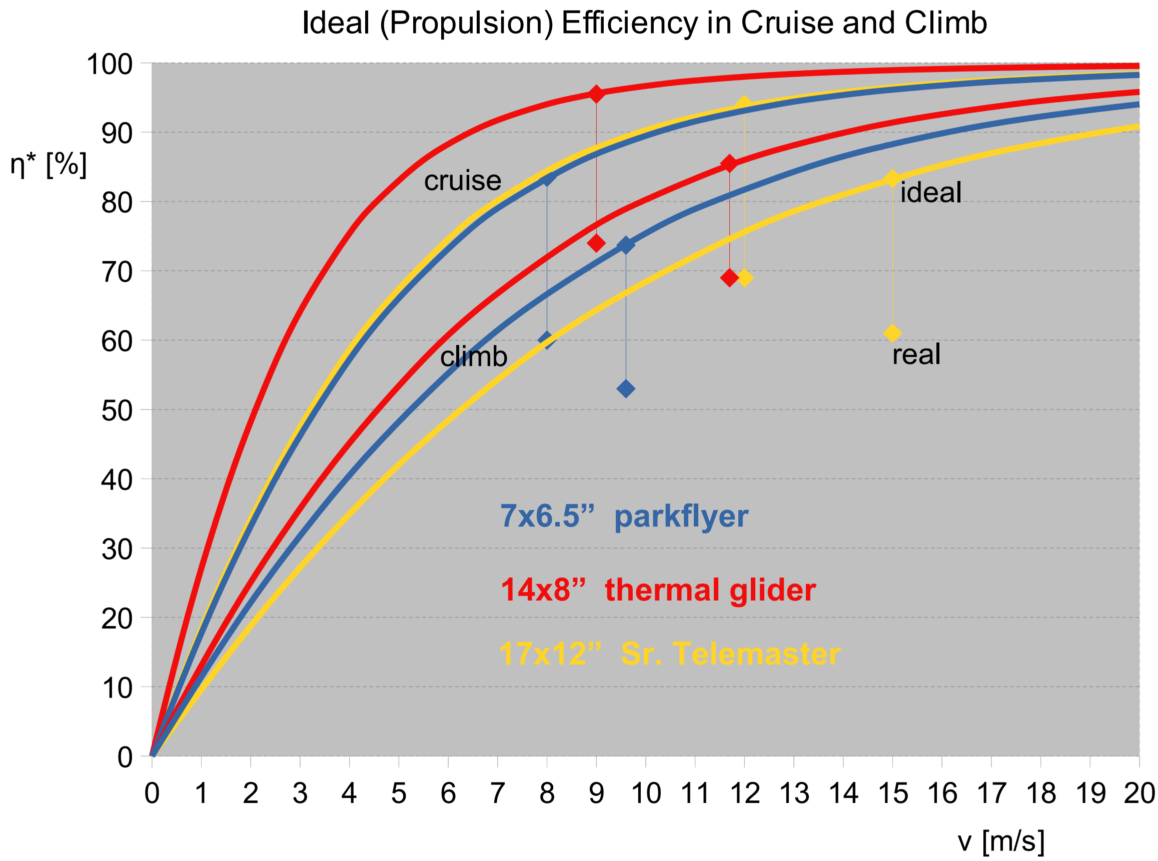
The most obvious difference between the propellers is their size, that is their diameter and disk area, which is basically diameter squared. Disk area relation is about 1:4:6. While a bigger propeller is more "powerful" than a smaller one, their power *loading* (shaft power per disk area) depends on the airplanes they are used on. The first and third pull draggy airframes which make for a similar power loading in cruise. In climb, the third is loaded more than the first because its drive is relatively more powerful. The second propeller is lightly loaded due to small drag in cruise and low weight in climb. Both, as well as a relatively big folding propeller, is typical for gliders.

All three propellers are close to their maximum efficiency in cruise but a few percent-steps below it in climb. The first and third are good for long cruise flights and short climbs, what is typical for their airplanes. The second, which has the smallest pitch to diameter ratio, is good for climb but still good for cruise due to the glider's low speed and low drag, which makes for an extremely low power loading.

In his Web page "How a Propeller Works", Martin Hepperle presents an equation combining a propeller's flight speed and "ideal" efficiency. It is solely based on momentum theory, considering the momentum the propeller provides to the air mass flowing through it. Thrust is produced by this repulsion so the energy (or power) spent on it is always lost, regardless of more or less other losses in addition.

Using the equation, different *curves of equal power loading* are drawn in an "ideal efficiency" diagram. It shows only the most basic influence on a propeller's efficiency, the repulsion losses in its slipstream (or wash), and neglects even the corresponding rotational losses in its swirl, which are smaller. Blade number, area, and shape as well as twist and airfoil distribution are neglected all the more. All losses are taken into account in the calculated "real" values, though.

Martin Hepperle generally demonstrates that high power loading is tolerable only at high speed if reasonable efficiency is required. Here, the specific cases are compared, that is the three propellers in cruise and climb each, using the data known from the performance calculations. This diagram shows indeed that power loading in relation to speed is the determining factor for a propeller's efficiency:



The three "ideal efficiency" curves for the respective *cruise* power-loading are quite close to the diagram's upper left corner, meaning quite good efficiency at low speeds. Due to much higher power loading in *climb*, the respective "ideal efficiency" curves are closer to a diagonal, meaning lower efficiency even at higher speed. The calculated "real" efficiencies are marked below the "ideal" ones at their respective speeds.

The example propeller is a medium case in this comparison. The parkflyer's ideal *cruise* efficiency line is about the same as that of the Sr. Telemaster (which is actually some kind of big parkflyer) because their power loading is nearly the same. The parkflyer is comparatively less powerful, so – at full power – its power loading is lower and hence its ideal *climb* efficiency line is higher than that of the Sr. Telemaster.

In the standard propeller diagram above, the difference between real and ideal efficiency in cruise is somewhat bigger than in climb. In this "ideal efficiency" diagram, the difference is only slightly bigger in cruise because climb is flown at higher speeds where the ideal efficiency curves are flatter and the differences tend to be bigger.

That holds for both the parkflyer and the Sr. Telemaster. It even partly explains why the parkflyer's differences are smaller than those of the Sr. Telemaster, which flies at distinctly higher speeds. Actually, the parkflyer's differences should be even smaller for its slow speeds. However, its propeller's "cheap" design results in less real efficiency than a "better" design like that of the Sr. Telemaster's propeller.

But even if the parkflyer's propeller were "better" (smaller differences) it would be still less efficient due to its lower flight speeds. Only a bigger propeller, spinning slower for lower power loading, really helps efficiency. For instance, the APC 9x6 SlowFly propeller (now used in place of the 6.9x6.3" toy propeller) spinning at 3185 (4930) rpm in cruise results in 117 (360) W/m^2 power loading and 68% (60%) real efficiency.

The glider propeller is a special case here because it is so lightly loaded. Hence it is very efficient, anyway. Then, its real climb efficiency is quite close to the respective ideal efficiency because it is well-designed and – with its low pitch – better suited for climb than the other two propellers. Yet power loading in relation to speed is the predominant factor of efficiency even in this case since not only the propeller is lightly loaded but also the model flies significantly faster than the parkflyer in cruise and especially in climb.

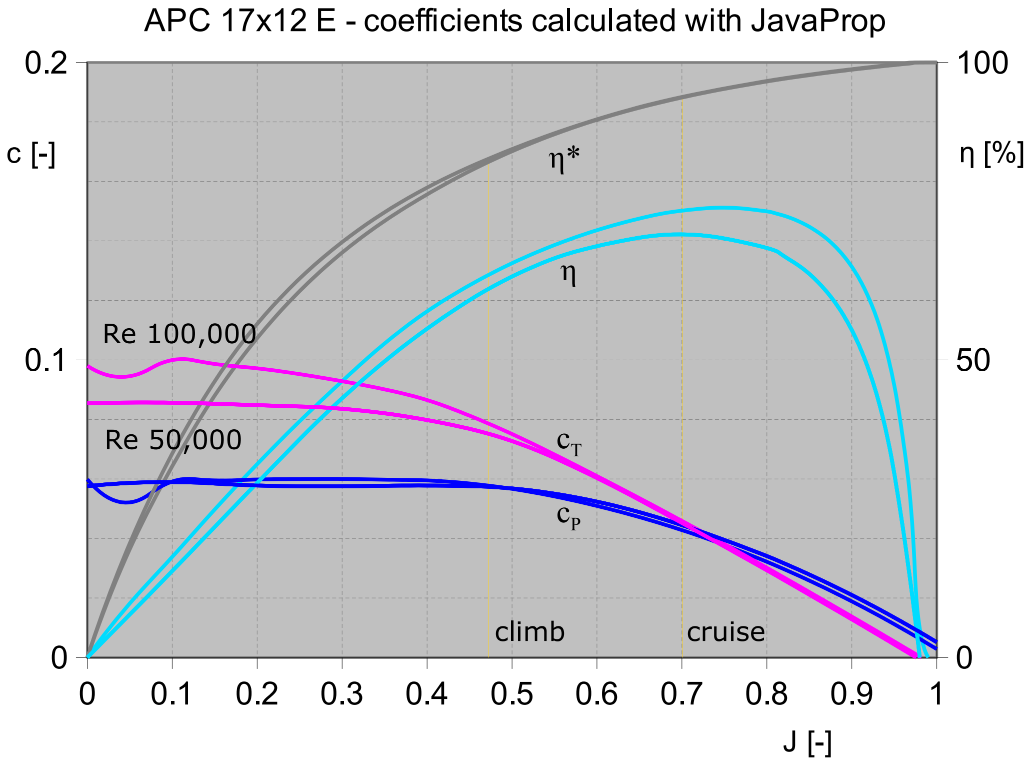
Coefficient Comparison

The example parkflyer propeller is so "cheap" that there is no other way to get coefficients than employing a calculation tool. Then again, for "better" propellers like the Sr. Telemaster's APC 17x12 E, the manufacturer calculated coefficients with a better tool, and there are even coefficients measured in a wind tunnel. That allows to compare them all and this way assess the accuracy and usefulness of JavaProp.

It is called "a relatively simple tool" by its author Martin Hepperle in his "Validation" Web page and the results are called "acceptable". We know from the performance calculations that this holds for the APC 17x12 E propeller.

Propeller coefficients are called "Performance Data" by APC and made available in the file "PER3_17x12E.dat". The calculation methods used and their limitations are mentioned in the "Engineering" Web page. They let us expect reasonable though still not perfect results.

Even the coefficients measured in the UIUC wind tunnel (by J. B. Brandt, M. Selig) can be not perfectly correct because no wind tunnel measurements are. Still these values (published in Volume 1, APC chapter, Thin Electric section) are deemed reliable here because measurements in different wind tunnels yield reasonably similar results (Propeller Measurements Comparison). So the coefficients measured at rotational speeds between 2000 and 3400 rpm are used as reference in comparison with coefficients calculated for 2000, 3000, and 4000 rpm.



In JavaProp, coefficients for 2000, 3000, and 4000 rpm were calculated using airfoil lift and drag coefficients for the Clark Y airfoil at the blade root and the ARA D 6% airfoil otherwise. First, airfoil coefficients for the Reynolds numbers 25,000 at root and 50,000 otherwise were used and then, in a second set of calculations, those for the Reynolds numbers 50,000 at root and 100,000 otherwise. These airfoil coefficients are prepared in JavaProp and were chosen as best fit to the propeller.

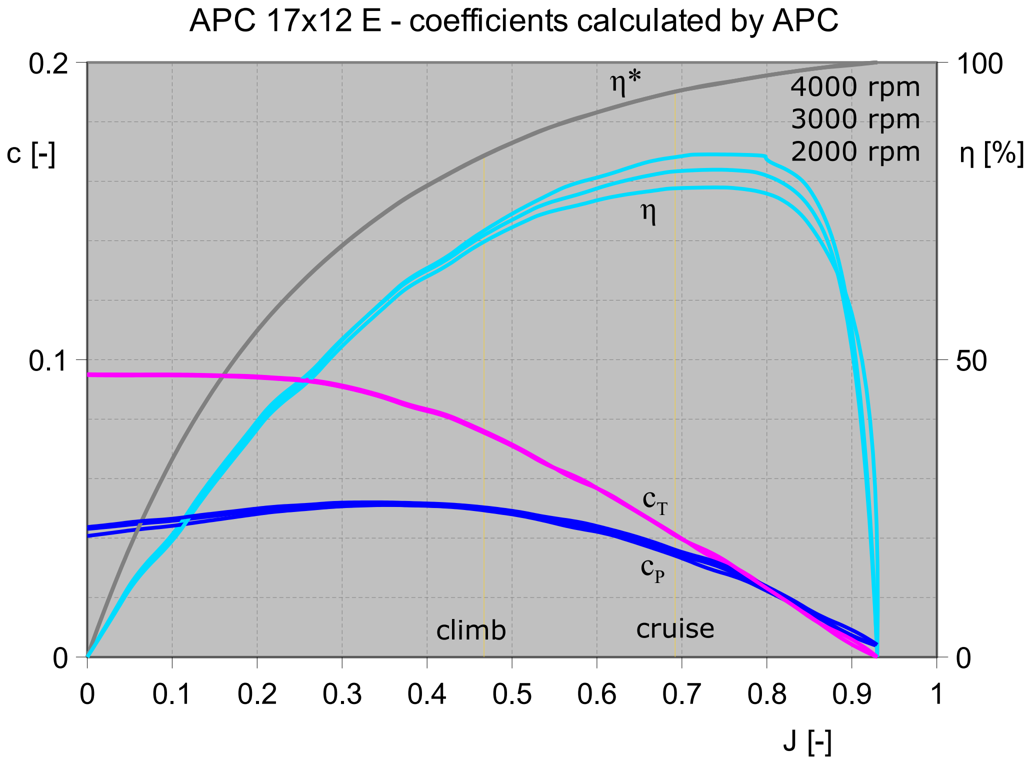
All calculated propeller coefficient values turned out the same regardless of rotational speed. The respective lines in the diagram are equal. Only different airfoil coefficients made a noticeable difference in propeller coefficients. Coefficient values for higher Reynolds (Re) numbers – matching higher rotational speeds – made for very slightly more thrust, slightly less power, and noticeably better efficiency.

The thrust coefficient c_T curve is typical for this simple calculation tool (which neglects complex aerodynamic effects) and well-designed propellers (which have little to no blade stall): It is roughly a straight line at high advance ratios and comes close to horizontal at low ones. For higher Re numbers, the values are only very slightly higher at high advance ratios while they are distinctly higher at low ones.

The power coefficient c_P curve is typical as well in that its high-advance-ratio part is an inverted parabola and its low-advance-ratio part is a horizontal line. The values for higher Re numbers are slightly lower at high advance ratios and even slightly higher at low ones.

Both the c_T and c_P curves for higher Re numbers are indented below 0.1 advance ratio. That is typical for JavaProp calculations and a manifestation of substantial but random blade stall predicted for low advance ratios (hence high power loading). This well-designed APC propeller shows far less of such indentations (blade stall) than the simple-design example toy propeller, even at the high power loading in climb.

At higher Re numbers, the airflow is "better" so thrust is higher and power lower, at least at higher (operating) advance ratios. That makes for noticeably better efficiency, which is essentially the quotient of both coefficients after all.



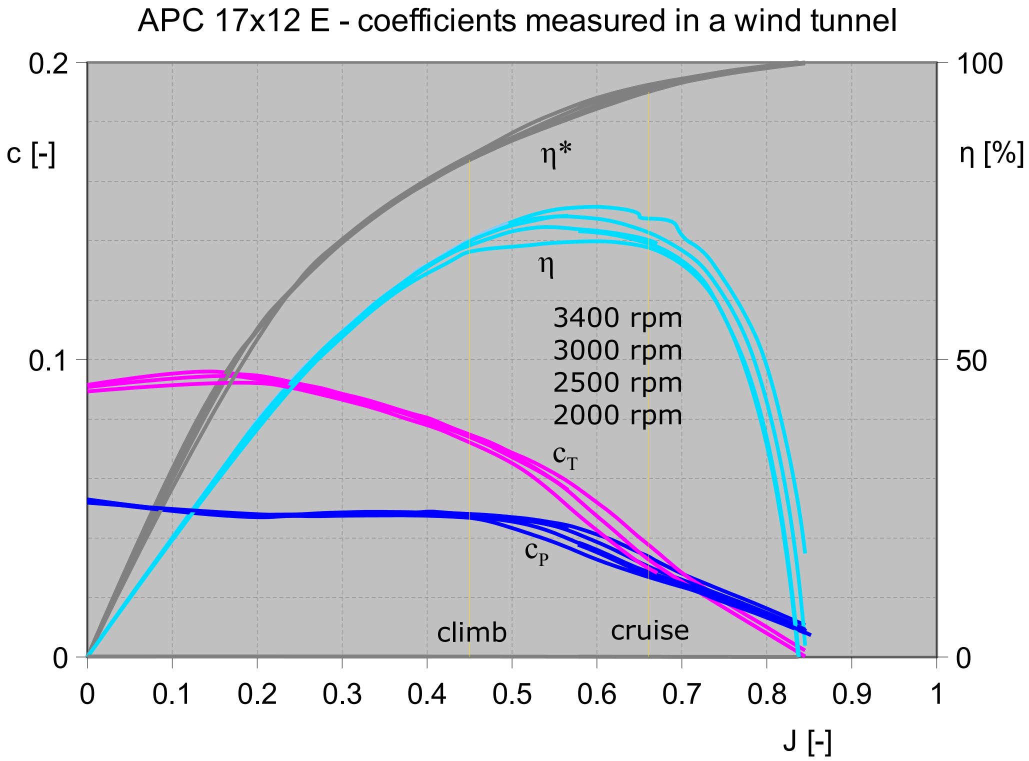
APC must have used Reynolds (Re) number dependent airfoil lift and drag coefficients in their propeller coefficient calculations. They seem to have incorporated more complex aerodynamic effects as well. At least, the respective coefficient curves are slightly different, like those for 2000, 3000, and 4000 rpm compared here, and they are not quite as schematic as curves calculated by JavaProp.

The thrust coefficient c_T curve is still nearly straight at high advance ratios and comes close to horizontal at low ones. The power coefficient c_P curve is still like an inverted parabola at high advance ratios but now it is sloped upwards at low ones. There are no indentations (random blade stall effects) at all and the curves differ only slightly also in the low-advance-ratio range.

Consistently, thrust coefficients are slightly higher at higher rotational speeds (Re numbers) while power coefficients are slightly lower. Consequently, efficiencies are noticeably better (higher) especially in the advance ratio range in which the propeller is operated (0.5 to 0.8).

Compared to the JavaProp calculation results, lower power coefficients c_P over the whole advance ratio range let the efficiency η curve come closer to the propulsion efficiency η^* curve. After all it is more efficient to produce about the same thrust with less power. And *peak* efficiency η is not only higher but also occurs at a higher advance ratio even though the maximum advance ratio is lower. Actually, that is distinctive of "better" propellers, but here the calculation tool used by APC just lets the propeller "look better" than JavaProp does.

Actually the curves look smooth but they are a bit wavy at closer inspection. That is probably due to the fact that the specified coefficient values have only three significant figures (while they have five in JavaProp). The wave amplitude is bigger than the difference between the curves, but the waves are equal in all curves so these can be still compared.



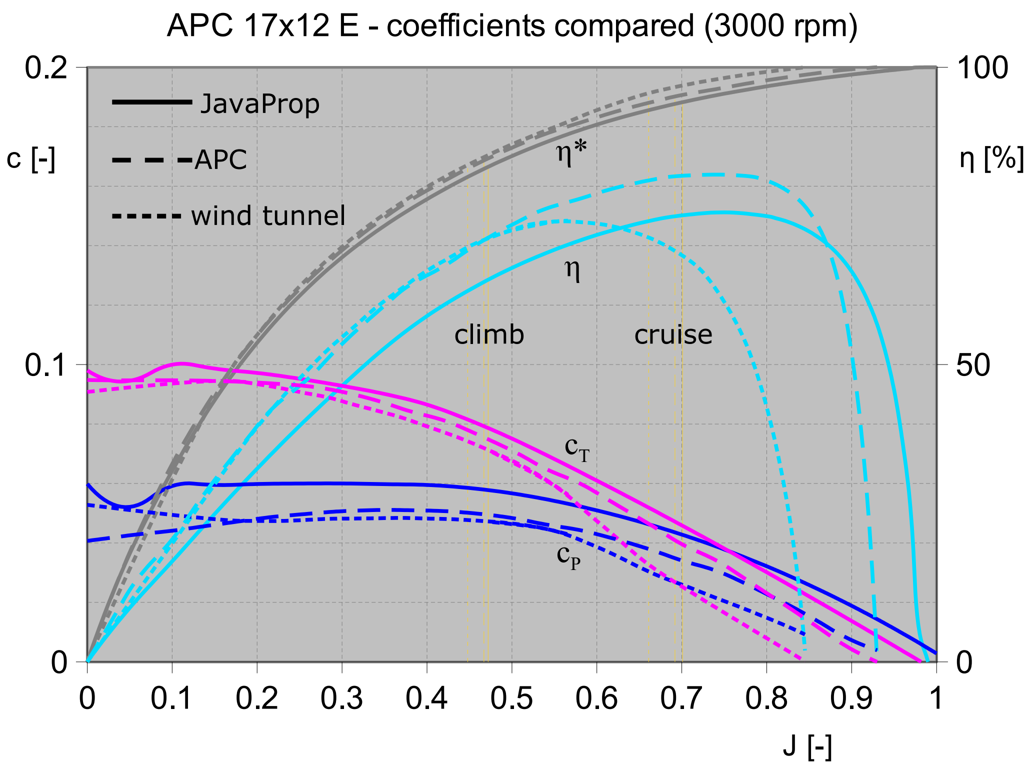
The coefficients measured in a wind tunnel (at 2000, 2500, 3000, and 3400 rpm) show the problem's complexity. The coefficient curves are smooth but *not* a straight line or an inverted parabola. They have even four parts: [1] 0 to 0.25 advance ratio, [2] 0.25 to 0.45, [3] 0.45 to 0.72, and [4] 0.72 to 0.85. Each part has its own slope and curvature so they have to be considered separately.

The curves are different for different rotational speeds (Reynolds numbers), but now *both* the thrust coefficient c_T curve and the power coefficient c_P curve are higher for higher rotational speeds. Still, thrust increase with Re number is higher than power increase so efficiency increases as well.

Obviously, Re number matters most in the third part of both coefficient curves since the biggest differences are there. In this advance ratio range the propeller is working in cruise flight. In the fourth part, at the highest advance ratios, the propeller is not operated so the noticeable differences in both curves don't actually matter. Yet it is worth noting that higher coefficient values also mean a shift to the right, to higher advance ratios. Hence, in the third and fourth curve parts, the efficiency curves are higher and more to the right and peak efficiency is at higher advance ratios.

In the second part, relevant for climb, there is a bit more thrust at higher Re numbers without actually needing more power. There is little increase of efficiency but it is not close to maximum, anyway, and does not really matter for climb. In the first part, relevant for take-off, efficiency is meaningless but a bit more thrust at higher Re numbers may be helpful.

All in all, measured coefficients seem to make for better accuracy and validity of drive calculations. Even though they have been measured only for rotational speeds up to 3400 rpm, they are still usable because the climb case (4500 rpm) is in the second part where the curves differ only faintly. The values measured at 2500 rpm are suitable for cruise, which is in the third part where the coefficient values are significantly different depending on rotational speed (Reynolds number). They could be even used for the climb case without much loss of accuracy.



The three ways to come up with propeller coefficients are compared by the 3000 rpm case. The respective data sets are distinguished by line style in the diagram.

The solid lines show values calculated by JavaProp, based on airfoil data for Reynolds number 100,000. Both coefficients c_P and c_T are remarkably high. That might be due to JavaProp inherently over-estimating power and thrust, the used airfoil coefficients being optimistic, the zero-lift angle-of-attack being wrong, and the numeric representation of the propeller's geometry being inherently incorrect.

Anyway, the dashed lines – showing the values calculated by the propeller's manufacturer APC with a better tool and based on correct geometry data – are notably lower. However, these seem to be still over-estimated because the values actually measured in a wind tunnel – the dotted lines – are even lower.

Lower coefficient lines mean lower maximum advance ratios and efficiencies η shifted to lower advance ratios. Most notably, the measured peak-efficiency advance ratio is considerably lower than calculated by both tools. The tool used by APC exaggerates efficiency while JavaProp over-estimates both coefficients in a way that peak efficiency is almost like measured. The related advance ratio is too high, but not even higher than calculated by the tool used by APC. This comparison illustrates that more pitch or advance ratio, respectively, provides the *potential* for better efficiency.

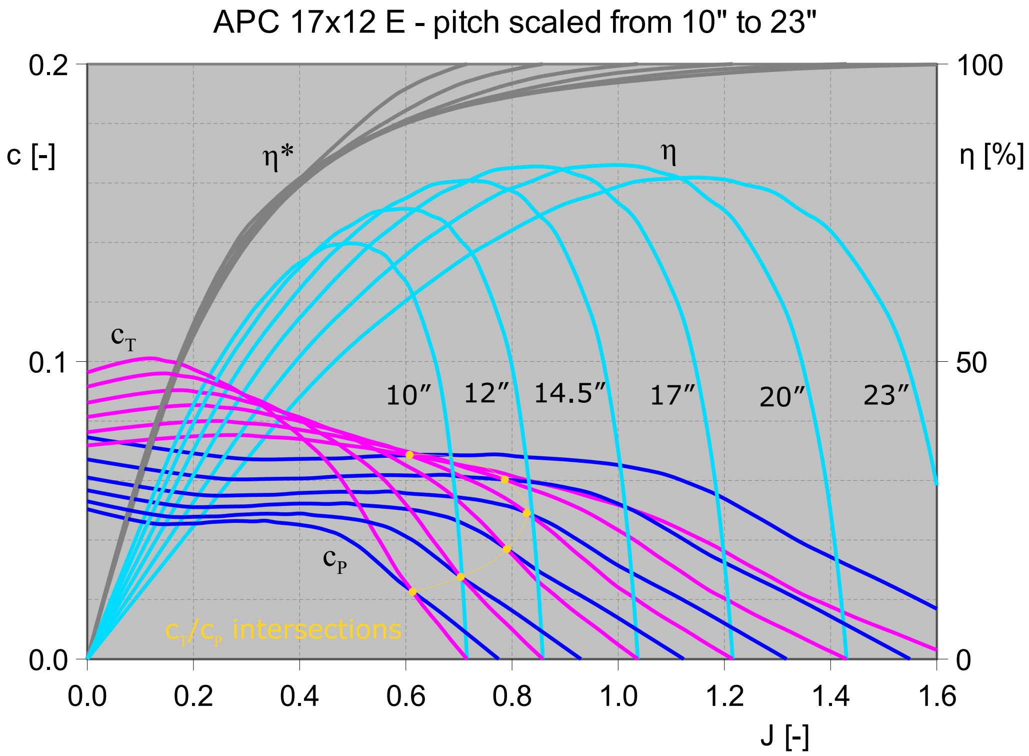
It also illustrates how the calculation tools more or less over-estimate coefficients or advance ratios, respectively, but under-estimate Reynolds number effects. For each set of coefficient values compared here, given thrust and airspeed values in climb and cruise, respectively, result in different advance ratios in the performance calculations. The higher the coefficient values are, the higher are the advance ratios.

Anyway, in case of the APC 17x12 E propeller, comparing the three options in the performance calculations showed that measured coefficients gave not even more realistic results overall. There seems to be no ideal way. Then again, imperfect coefficient values is yet another deficiency that is of relatively little relevance in our calculations because power loading and speed are the most important factors.

Efficiency

The APC 17x12 E is a "better" propeller and there are reliable coefficient values measured in a wind tunnel, even if for one pitch (12″) only. But it is possible to make an educated guess on how the coefficient values are modified by varying pitch and then see the implications for the propeller's efficiency. This is just a simple experiment:

The propeller's 12″ pitch is scaled to 10″, 14.5″, 17″, 20″, and 23″. It stands to reason that the c_T and c_P coefficient curves are scaled by the same ratio horizontally, in the direction of the J-axis. Higher pitch (steeper blade angles) makes for more blade stall at low airspeed (low advance ratio J) and hence less thrust and more power demand. To represent that, the c_T and c_P curves are simply scaled down or up, respectively, but (arbitrarily) by only 30% or 31.5%, respectively, of the pitch scaling.

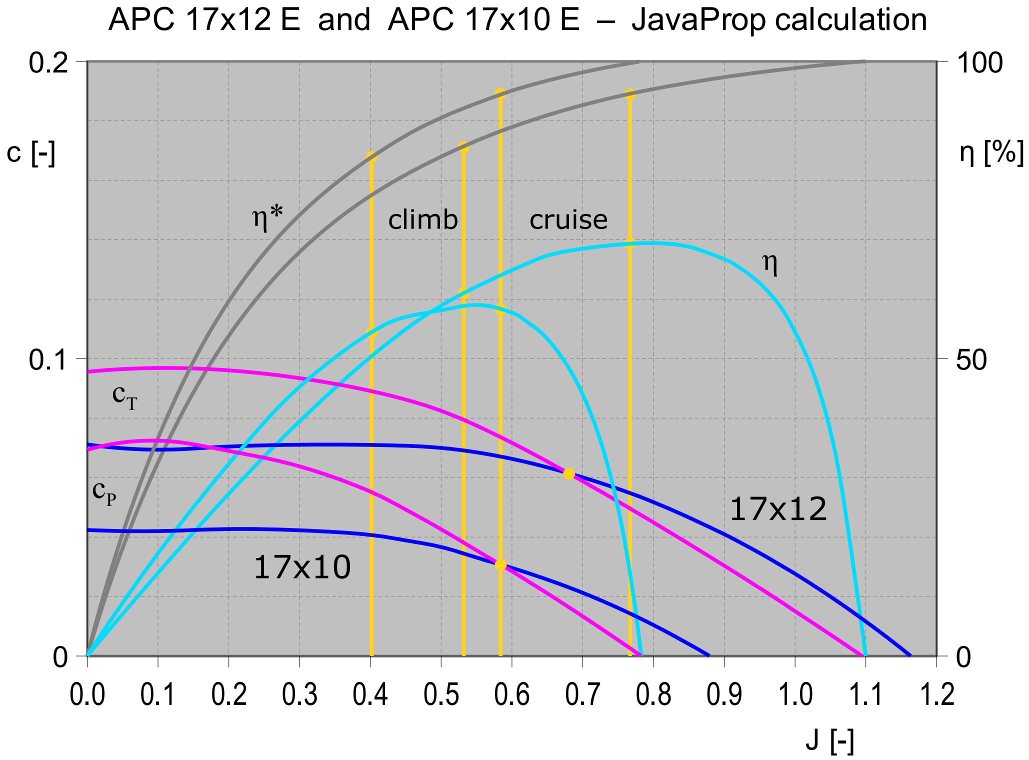


This array of coefficient curves seems reasonable and – more important – typical. The efficiency η and propulsion efficiency η* curves are calculated from the scaled coefficient curves. Of course, a "square" 17x17 propeller would be excessive in practice, not to mention a 17x20 or even 17x23. They would have lower peak efficiencies and these at lower advance ratios than shown here. But this *theoretical experiment* just goes to *illustrate* that even a "perfect" propeller can have not more than 83% peak efficiency, and that only at 1.0 advance ratio, what requires excessively high pitch (17x20).

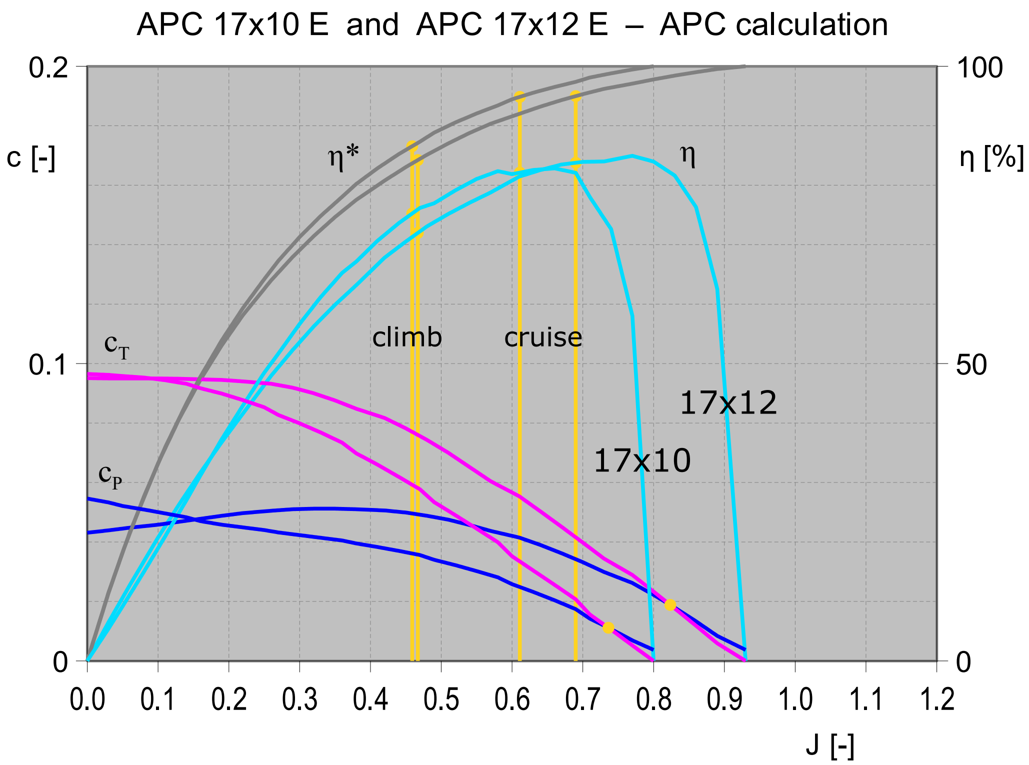
In practice, a pitch-to-diameter ratio higher than the APC 17x12 E's 0.7 would hardly bring higher efficiency even with a "better" propeller like this one. But then again, the APC 17x**10** E as well as the 8″, 7″, and 6″ fine-pitch 17″-diameter variants actually have *lower* peak efficiencies. So the highest pitch-per-diameter variant available seems superior – for high flight speed and cruise, and even for climb (but not for 3D).

A naive explanation: The c_T and c_P curves intersect somewhere. Efficiency η is their ratio times the advance ratio J, so it is equal to J at the intersection. The closer this is to the maximum advance ratio, the more is maximum η limited by low J "on its left".

Or even simpler: They are in the J range where propulsion efficiency η* is low. This is too simplistic, though, and we need to know the operational cases *climb* and *cruise*.



By a "quick and dirty" calculation with PropellerScanner and JavaProp, two 17" propellers with different pitch had been compared for the Sr. Telemaster. Coefficient values as well as advance ratios are even more exaggerated than in the previous section.



The same two propellers have been compared again by applying the coefficient values calculated by APC, which are closer to reality (even if the curves are somewhat wavy).

As we know from the previous section, JavaProp over-estimates the coefficient values and especially the advance ratios in a way that efficiencies are *not* over-estimated but just at too high advance ratios. The c_T and c_P curves intersect at rather low advance ratios and the two efficiency η curves are distinctly apart from each other and hence have clearly different peak values.

Then again, the coefficient calculations done by APC just slightly over-estimate advance ratios but in a way that efficiencies are *both* over-estimated and at too high advance ratios. The c_T and c_P curves intersect at rather high advance ratios and the two efficiency η curves are close to each other and have nearly equal peak values.

The respective climb and cruise advance-ratios are known from the performance calculations. In all four cases (both propellers, both coefficient calculation tools), *cruise* flight speed had been given as 12 m/s, to be achieved by an appropriate power setting. As a matter of course, there is virtually the same thrust as well as the same propulsion efficiency η^*. The lower-pitch 17x10 has to spin faster than the 17x12 because it gets only at lower advance ratios. The two propellers' cruise advance-ratios are closer to each other if the coefficients calculated by APC are used but they are yet in the same range in any case.

The *climb* flight speeds resulted as best climb speeds from the respective performance calculations. To compensate for the different pitches, a 5S battery was assumed for the 17x10 propeller but a 4S battery for the 17x12. If the coefficients calculated with JavaProp are used, the best climb speeds with both propellers happen to be equal (15 m/s) while – consequentially – the respective advance ratios are clearly different. The opposite if the coefficients calculated by APC are used: The best climb speeds are clearly different (17.5 m/s or 14 m/s for the 17x10 or 17x12, respectively) while – again consequentially – the advance ratios are nearly equal. Again, the two propellers' climb advance-ratios are yet in the same range.

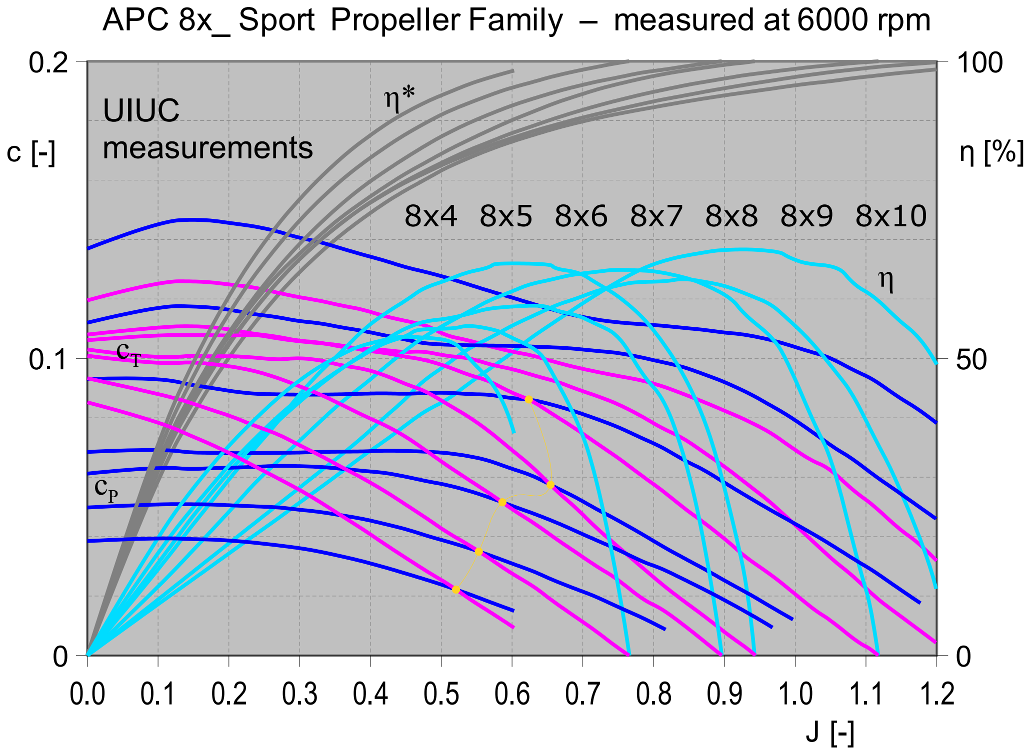
If the coefficients calculated with JavaProp are used, the 17x12 propeller is clearly more efficient than the 17x10, both in cruise (69%/58%) and climb (61%/54%). If the coefficients calculated by APC are used, the 17x12 is only slightly better in cruise (84%/82%) and even less efficient in climb (72%/75%). In view of the previous comparisons, the efficiency values in the first case (JavaProp) are probably close to reality while they are far too high in the second case (APC). The respective advance ratios are too high in the second case and far too high in the first.

All that makes it virtually impossible to predict which propeller would give the best efficiencies on the Sr. Telemaster. A mere guess is that reality could be in between, a compromise or blend of the two cases: Advance ratios clearly lower than in both cases and efficiency values like in the first case would result in only *slightly* higher efficiencies for the 17x12 propeller in both cruise and climb. That in turn would mean the efficiency difference is of little relevance in practice and the 17x12 may be just the best propeller with a 4S battery and the 17x10 with a 5S.

There is no way to bring this to an issue since there are no wind tunnel measurements for the APC 17x10 E propeller. There is only an analogy, that is several propellers of same shape and diameter but different pitch, which have been measured in a wind tunnel. Not knowing any operational climb and cruise cases for them, and considering that coarse- and fine-pitch propellers are meant for different applications, we can merely see how the respective efficiency η curves compare to each other.

(For information about the measurements see: Brandt, J.B. and Selig, M.S., "Propeller Performance Data at Low Reynolds Numbers", 49th AIAA Aerospace Sciences Meeting, AIAA Paper 2011-1255, Orlando, FL, January 2011.)

Propellers of same diameter but different pitches are loosely called a "propeller family" here. That with the most members (pitches) measured at UIUC (by J. B. Brandt, M. Selig) is the APC 8x_ Sport. Their design is different from that of the APC E (Electric) propellers but the measured coefficient curves' shape is equally complex. We compare them to each other (family members) only, so this comparison seems meaningful:



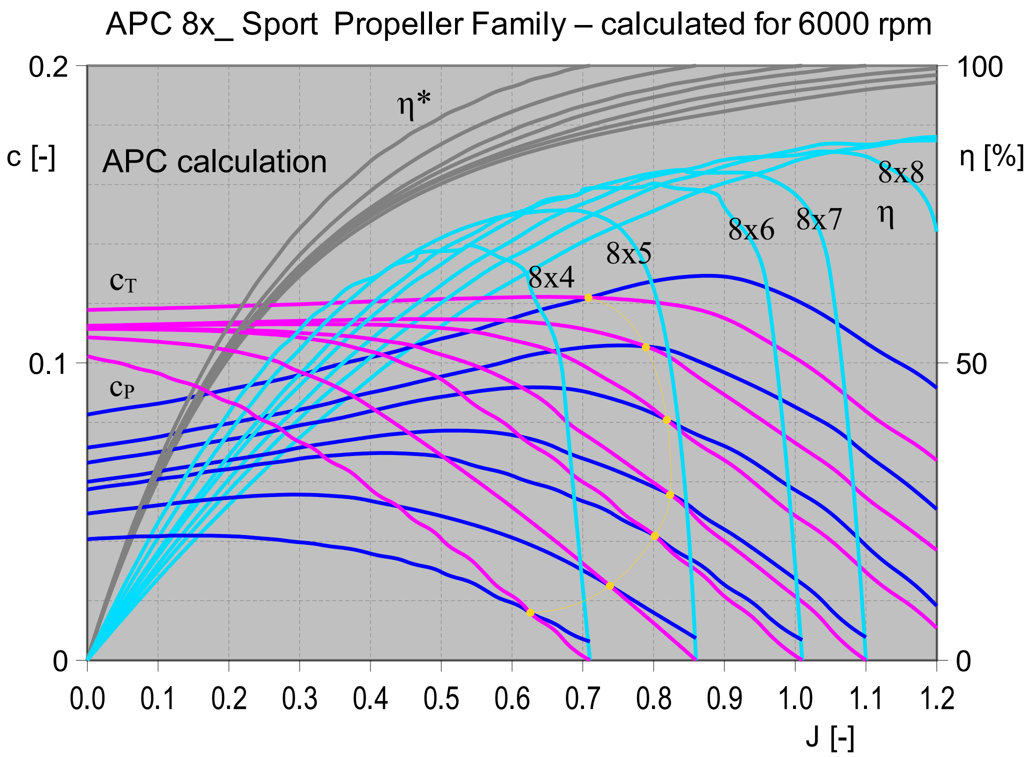
In practice, the c_T curves do not intersect like in the theoretical experiment above. There is an irregular pattern of c_T and c_P curve intersections in that the 8x7 deviates and there is no such intersection in case of the 8x9 and 8x10 propellers.

The pattern of efficiency η curves is irregular as well. The 8x4, 8x5, 8x8, and 8x10 have higher and higher peak efficiencies at higher and higher advance ratios, as would be expected. Indeed there is a peak-efficiency advantage of coarse pitch over fine pitch. But the 8x7 has higher peak efficiency than the 8x8 yet at the same advance ratio as the 8x6, and even relatively high efficiency at lower advance ratios – that is exceptionally good. The 8x6 and the 8x9 have their respective efficiency peaks at the expected advance ratios. But the 8x6 has too low efficiency and the 8x9 has even lower peak efficiency than the 8x8. Only the 8x10 duly has the highest peak efficiency at the highest advance ratio, which is not even 1, though.

The differences in peak efficiency over a propeller family's pitch range, exceptionally efficient as well as less efficient members of a propeller family, the lower efficiencies of the 8" Sport propellers compared to the 17" Electric propellers – all this could be more or less predictable by the calculation tools. To check, both tools are compared to measurement.

Again, we implicitly assume the measured coefficients to be close to reality so they are used as reference. Conversely, that means the exceptions – especially the 8x7 – are just that but not aberrations, or errors in measurement. This is supported by the fact that the measured coefficient curves are consistent and by the long-standing knowledge that such exceptions, or variations exist in propeller families.

APC published "performance data" for their APC 8x_ Sport propeller family. These are calculated with an advanced tool so they should be a good approximation to the real coefficient values. Yet the tool produces significant deviations from the measured coefficient curves' shape, especially the c_P curves', and it is not able to identify any "exceptions" in a propeller family:

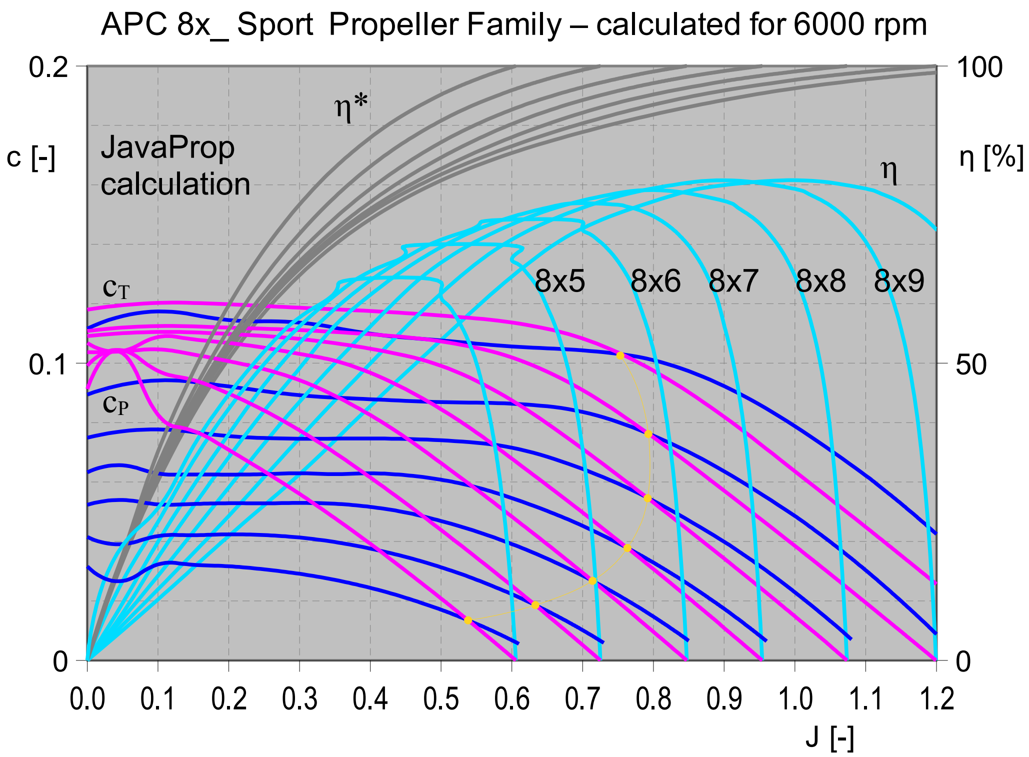


This is a consistent array of coefficient curves with a regular pattern of c_T and c_P curve intersections, as would be expected from a calculation tool. That makes for consistently higher and higher peak efficiencies at higher and higher advance ratios – without exceptions.

Like in the cases before, efficiency η and advance ratio J are overestimated in APC calculations. The efficiency η curves come close to the propulsion efficiency η^* curves, hence differences in efficiency are exaggerated.

The c_T curves somewhat resemble those in the previous diagram while the c_P curves are sloped upwards towards higher advance ratios J. That has been seen before in the previous section but there is no obvious explanation.

Since geometry data are published by APC it is possible to calculate coefficients with JavaProp, which is a simpler tool but may even yield better results than the APC calculations in that it does not overestimate efficiencies and their differences as well as it does not provide c_P curves that are sloped upwards. But it is not able to identify any "exceptions" in a propeller family, either:



This is again a consistent array of coefficient curves with a regular pattern of c_T and c_P curve intersections that makes for consistently higher and higher peak efficiencies at higher and higher advance ratios – without exceptions.

Unlike in the cases before, JavaProp does not overestimate the advance ratio J here but even slightly underestimates it instead. The efficiency η curves are overestimated now because the power coefficients c_P are underestimated.

The c_T curves as well as the c_P curves are indented at low advance ratios J, what is typical for JavaProp (presumably due to predicted blade stall). There are "waves" in the c_P curves of the coarse-pitch variants that are similar to those in the measured curves so this seems indeed more realistic than the APC calculations.

To sum up, a rule "higher pitch means higher peak efficiency" would pertain but there may be "positive" and "negative" exceptions which render it useless. The rule would apply to high-speed and cruise flight, but not necessarily to climb. The differences may be small. Fine-pitch propellers with low peak-efficiency are anyway meant for 3D and similar applications, and coarse-pitch propellers are meant for special applications (speed) as well. Then, efficiency depending on pitch is of relatively little relevance in our calculations because design and size (diameter), power loading, and speed are the most important factors. There seems to be no reliable way to identify the most efficient member of a propeller family without measured coefficient values for all of its members. So peak efficiency is not a viable selection criterion.

Propeller Pitch

Pitch is a general, context-dependent term. In the section Propeller Example, it was used for the most basic definition, that is local pitch H. A propeller's nominal pitch H_n is often specified as the local pitch at 75% of the blade radius R. This is a "geometric" definition meaning the distance traveled by the propeller in one turn if it were a perfect "air screw" and the blade airfoil's chord line would represent its thread pitch.

There is an "aerodynamic" definition as well. Since thrust is generated by repulsion, there is always some slippage, except when no thrust is generated at a certain advance ratio. This in turn is actually the ratio of a flight speed and a circumferential blade-tip speed but is colloquially called "pitch speed". Not only this characteristic speed is known from the performance calculations but others as well. They can be converted into the distance *really* traveled in one turn and that is called pitch as well:

Type of Model	55″ retro parkflyer		100″ thermal glider		95″ Sr. Telemaster	
Propeller	Günther 6.9x6.3″		aero-naut 14x8″		APC 17x12 E	
Pitch in inches:						
nominal	6.3	96%	8.0	98%	12.0	81%
measured at 0.75·R	6.6	100%	8.2	100%	14.8	100%
at "pitch speed"	5.8	88%|100%	9.7	119%|100%	16.7	113%|100%
at maximum speed	4.5	69%|78%	9.1	111%|94%	13.6	92%|82%
in cruise	3.9	59%|67%	7.9	97%|81%	13.1	88%|79%
in climb	3.1	47%|54%	5.7	70%|59%	9.0	61%|54%

The pitch at 0.75·R used here is as "measured" by PropellerScanner from front and side views of the propeller and displayed in JavaProp. It is used as reference (100%) because the propeller coefficients have been calculated from the measured geometry and, in turn, the performance calculations are based on them.

In case of the parkflyer propeller, and the glider propeller as well, the measured value fairly conforms to the specification (nominal pitch). Obviously, measuring the Sr. Telemaster propeller's geometry did not go well as its measured pitch is considerably bigger than the nominal. Still the performance calculations are based on it.

The parkflyer propeller with its flat-plate airfoil would be expected to have zero thrust when traveling at its measured (geometric) pitch since the blades would work at zero angle-of-attack (AoA) then. Its real (aerodynamic) pitch is smaller, though, because its local pitch *distribution* is disadvantageous. The other two propellers are "better" as they feature both a suitable local pitch distribution and more efficient, cambered blade airfoils (which have a negative zero-lift AoA). Hence their zero-thrust pitch is substantially bigger than their nominal or even measured pitch.

This zero-thrust pitch, also called a propeller's "aerodynamic pitch" here, is used as an additional reference (100%) for a pitch comparison in descending order. At maximum straight-and-level flight speed, the propeller is working at high rotational speed and low blade-AoA. In cruise flight, rotational speed is low and blade-AoA is higher. In climb, blade-AoA is even higher – although rotational speed is high – because thrust is big. The higher blade-AoA, the higher is slippage and the lower is real pitch.

This *order* of pitches is common to all propellers, but the relative values can be quite different. A propeller's aerodynamic pitch may be even higher than its geometric pitch (second and third propeller) but it may be lower as well (first propeller).

A "better" designed propeller like the second, which is moreover operated at relatively low power loading and high flight speed, has relatively high aerodynamic pitches overall. The third propeller, which is "better" designed as well, is operated at even higher flight speed but substantially higher power loading, so its relative aerodynamic pitches are not quite as high. The first propeller is operated at slow flight speed and substantial power loading, and is a "cheap" design, which is why its relative aerodynamic pitches are the lowest overall.

The higher a propeller's efficiency – due to design as well as power loading and flight speed – the higher are relative pitches. In the end it seems that, depending on efficiency, aerodynamic pitch can be 10% to 20% higher than geometric pitch. Then, not only aerodynamic "pitch speed" but even maximum level-flight speed can be higher than geometric "pitch speed", but only with a slick airplane and not with a draggy one. That holds even independent of the propeller-coefficient calculation's quality.

There have been several deficiencies in the process of calculating the coefficients for the APC 17x12 E propeller. The front and side view photos are obviously inexact. Processed with PropellerScanner, they result in even 14.8″ pitch at 0.75·R while the really good pictures made at UIUC result in only 13.6″. Then, there was a systematic fault in applying PropellerScanner as the geometry "digitized" at UIUC shows only 12.5″ pitch at 0.75·R what is close to the nominal value. In JavaProp, which is a simplified tool already, the ARA D 6% blade airfoil at Reynolds number 100,000 has been used. All that made for exaggerated geometric as well as aerodynamic pitch values:

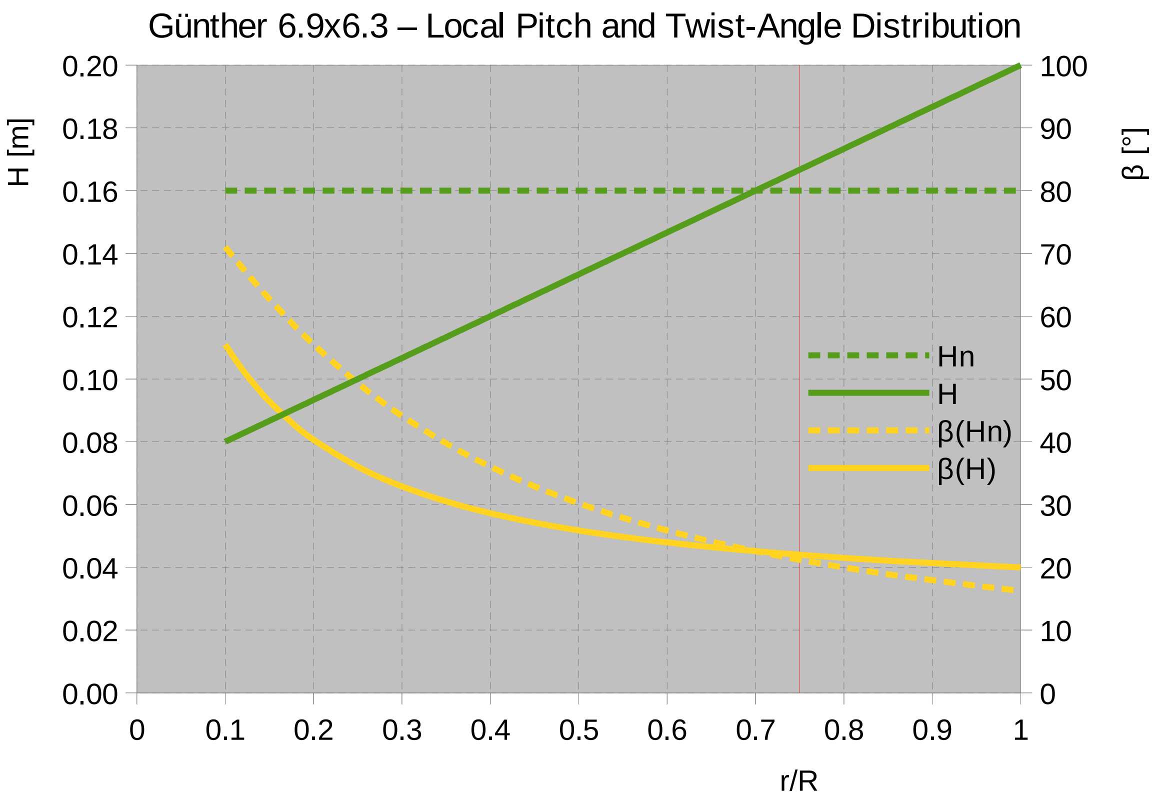
APC 17x12 E	UIUC "digitized" geometry		PropellerScanner from photos	
	wind tunnel measurements		JavaProp calculations	
Pitch in inches:				
nominal	12.0	96%	12.0	81%
measured at 0.75·R	12.5	100%	14.8	100%
at "pitch speed"	14.6	116%|100%	16.7	113%|100%
at maximum speed	11.8	94%|81%	13.6	92%|82%
in cruise	11.2	90%|77%	13.1	88%|79%
in climb	7.7	61%|53%	9.0	61%|54%

The absolute values are indeed substantially lower if the performance calculations are done with measured propeller coefficients. Yet the difference between aerodynamic and geometric pitch is even bigger, and the *relative* values differ by only 0 to 3 percent steps.

This comparison of pitch values shows more sharply (than the Coefficient Comparison section) what to expect from imperfect propeller coefficient values: For the thrust needed at certain flight speeds, too low rotational speeds will be predicted by the performance calculations. Torque and even power values will be less affected.

Pitch Distribution

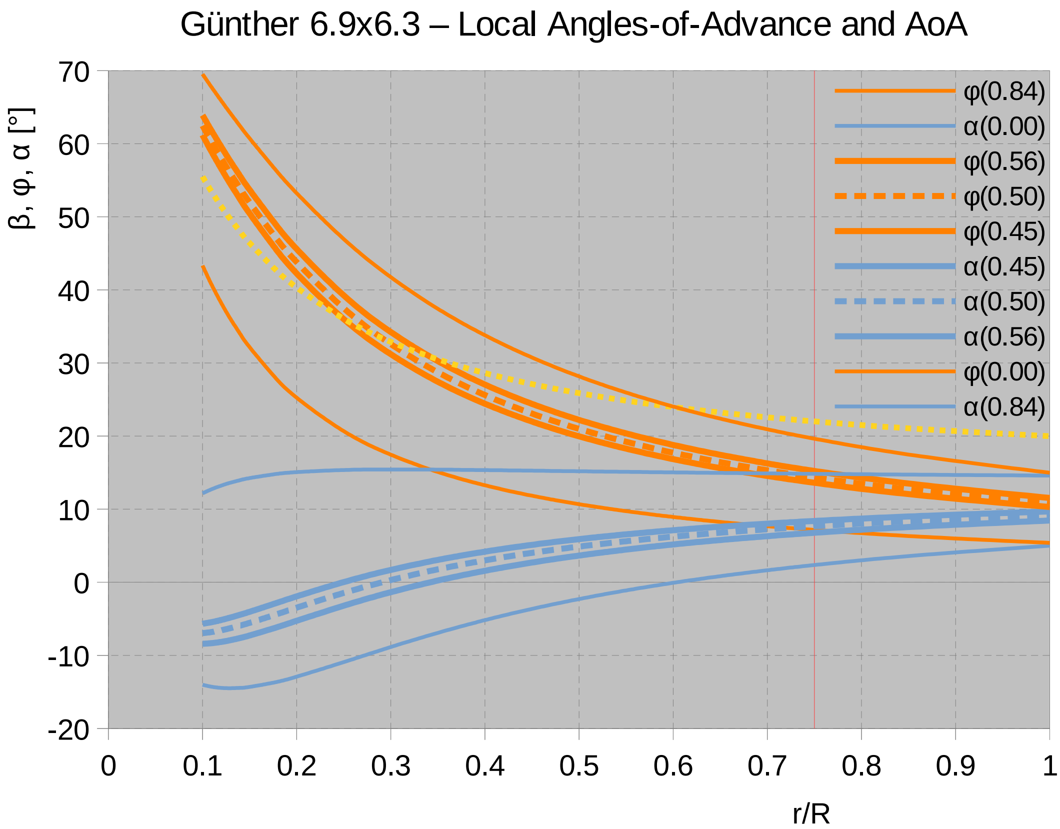
The "cheap" example propeller has elliptic blade planform and flat-plate blade airfoil. It has a peculiarly simple distribution of twist angles and pitch over the blade radius, called *local* twist angle and pitch, respectively. The PropellerScanner results (in the Propeller Example section) suggest a linear distribution of pitch H as shown here:



Only at $0.70 \cdot R$ (not at $0.75 \cdot R$ as usual) local pitch H is equal to the specified, or nominal pitch H_n. This pitch, constant over the whole blade radius, would result in corresponding twist angles $\beta(H_n)$, which are the local "advance angles" in case the propeller advances at its "nominal-pitch speed". Since the actual local pitch H is smaller below $0.70 \cdot R$ and bigger above, the actual local twist angles $\beta(H)$ are lower or higher, respectively, than the local advance angles at "nominal-pitch speed". In this case, there would be positive local blade angles-of-attack α above $0.70 \cdot R$ and negative below.

If the propeller advances at a certain advance ratio J, then the distance h (here called pitch as well) that it travels in one rotation is proportional (by factor J) to the distance that one blade tip travels circularly in this rotation. The ratio of these two distances is the tangent of the advance-angle φ, so this angle is the ratio's arc-tangent. Since the distance h is constant but the circular distance is proportional to the radius, the local advance-angles φ at inner radius locations are bigger than at the blade tip.

Correspondingly, the local twist angles $\beta(H)$ are a ratio's arc-tangent as well, now the ratio of the local pitch H and the distance traveled circularly at the radius location. Of course, the latter is again proportional to the radius but the former is not constant (like h) so the twist-angle distribution is even different from the advance-angle distribution in the "pitch speed" case. At any radius location r/R, the local angle-of-attack α is the difference between twist-angle $\beta(H)$ and advance-angle φ.



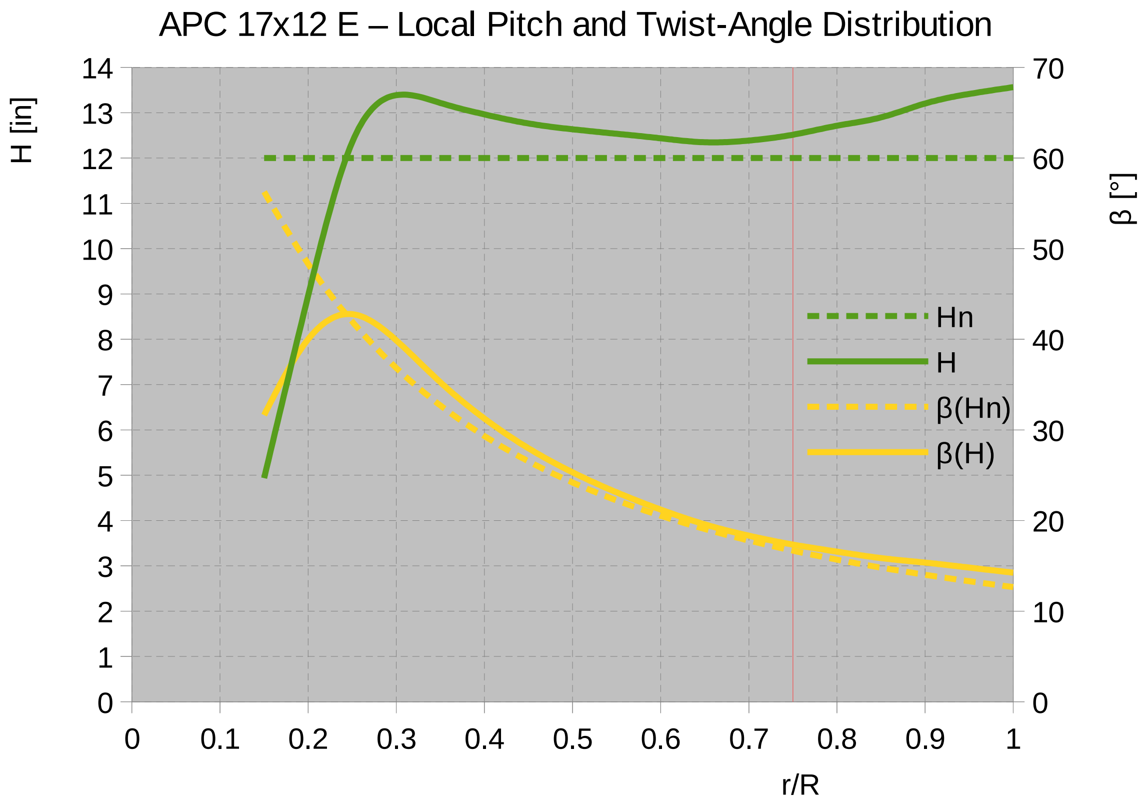
In this diagram, the twist-angle distribution $\beta(H)$ is repeated as a yellow *dotted* line to make it unobtrusive. It is used to subtract different advance-angle distributions φ for characteristic operating points, giving the respective angle-of-attack distribution α. The advance-angle distributions φ are calculated including an "induced advance ratio" to allow for an average slipstream speed. That makes for angle-of-attack distributions α which are at least closer to reality than some neglecting the slipstream at all.

This holds especially in the static case ($J=0.00$) in which the angle-of-attack α is virtually the same over nearly the whole radius. This is an approximation because slipstream speed is actually not equal over the radius (but smaller towards root and tip). Still it is interesting to see that, despite high "static" slipstream speed, α is even about 15°. That may explain why JavaProp indicated completely stalled blades at low advance ratios. Yet the propeller seems to be designed to the static case.

The opposite case, zero thrust or "pitch speed" ($J=0.84$), is just as remarkable in that the blade's most effective outer part ($r/R>0.6$) has still positive angle-of-attack α while the inner part has negative. JavaProp says that 22% of the blade are stalled and this must be its inner part. Obviously, positive and negative local thrust balance each other so that overall thrust T is zero. That is inefficient and the reason why this propeller's "pitch speed" is even 8% *lower* than its nominal pitch would suggest.

The climb ($J=0.45$) and cruise ($J=0.56$) cases delimit the narrow "band" in which the propeller is operated. Just for information, the dashed line delimits the even narrower band ($J=0.45$ … 0.50) in which no blade stall occurs (according to JavaProp). Angle-of-attack α is negative in the blade's not relevant inner 30% and increasingly positive towards the tip where it approaches 10°. That is below the flat-plate airfoil's stall limit so there is no blade stall. At higher advance ratios, there must be negative stall at the slower moving inner blade parts. There seems to be no obvious rationale.

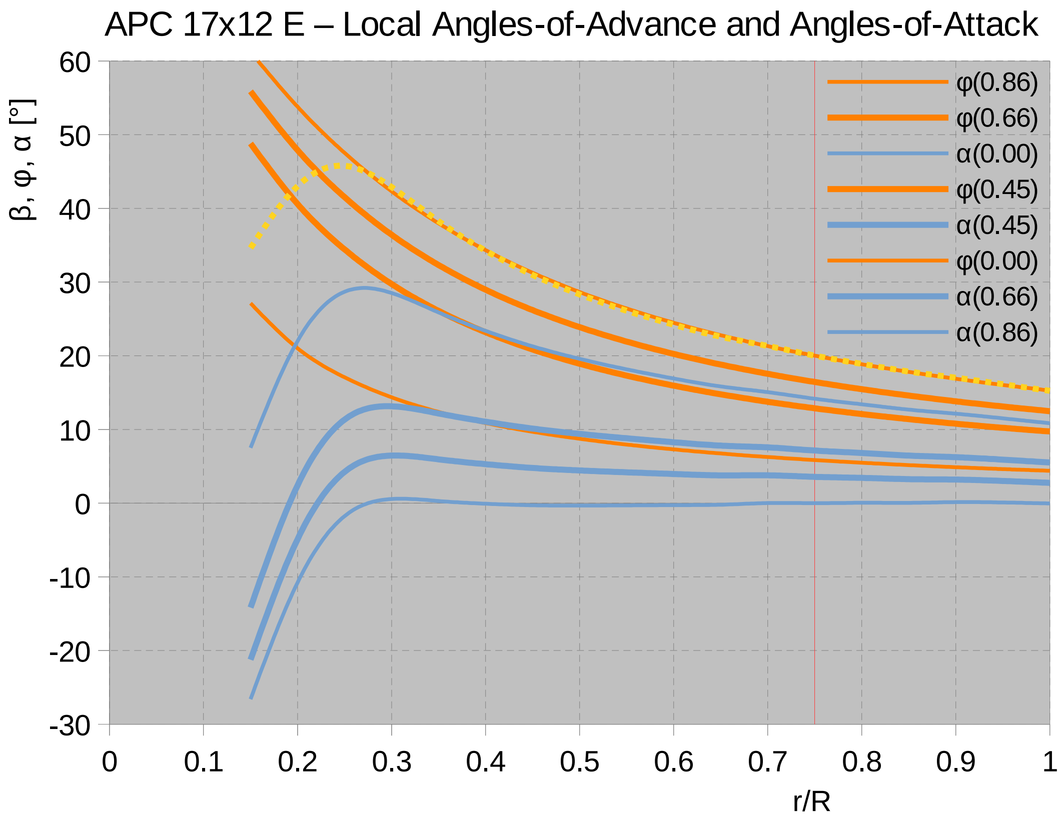
The "better" APC 17x12 E propeller has a complex blade planform and efficient cambered blade airfoils. Its geometry has been meticulously "digitized" at the UIUC. It shows pitch H and twist-angle $\beta(H)$ distributions close to the nominal pitch H_n and nominal-pitch twist-angle (or "nominal-pitch speed" advance-angle) $\beta(H_n)$ distributions, respectively:



Below $0.3 \cdot R$, pitch H and twist-angle $\beta(H)$ plummet to a low value. From a "kink" at $0.3 \cdot R$, the blade is "twisted back" until it reaches the hub. Presumably that is because the hub is relatively (to its diameter) thin so the blade must end there at a small twist angle. However, the blade's inner 30% are not relevant, anyway.

Nowhere in the outer, effective 70% is local pitch H equal to nominal pitch H_n, not at $0.75 \cdot R$ either. It is lowest at $0.65 \cdot R$ and increases towards $0.3 \cdot R$. As the difference between $\beta(H)$ and $\beta(H_n)$ suggests, this makes for an increasing geometric angle-of-attack α_{geo} and is probably intended. Towards the tip, H increases as well what is actually a geometric wash-*in*. Usually there is wash*out* towards the tip (to avoid tip stall) so there is no obvious rationale as far as only blade geometry is considered.

The cambered blade-airfoils' aerodynamic angle-of-attack α_{aero} has to be included in the consideration.



The local twist-angle distribution $\beta(H)$ is not just repeated here (as an overlay yellow *dotted* line) but is shifted up by the local zero-lift angle-of-attack α_0, which is negative for cambered airfoils. Due to this camber, the propeller advances "aerodynamically" faster than "geometrically".

At their "Engineering" Web page, APC just mentions blending different airfoils from root to tip or some washout near the tip, respectively. By experimenting, α_0=-3° was found to be a good value overall since it made the twist-angle distribution $\beta(H)$ virtually congruent with the advance-angle φ curve in the zero-thrust ("pitch speed") case, at least from $0.3 \cdot R$ to $0.7 \cdot R$. From $0.7 \cdot R$ to $1.0 \cdot R$, there was still geometric wash-in but it could be compensated by linearly reducing α_0 to -1° at the tip, what is equivalent to aerodynamic washout (less camber at the tip).

So possibly APC designed a constant "aerodynamic pitch" into the propeller, that is from $0.3 \cdot R$ to $1.0 \cdot R$ which is actually the effective part. In the "pitch speed" case (J=0.86), the aerodynamic angle-of-attack α is all zero as a consequence. In cruise flight (J=0.66), α is all positive but bigger towards the root where circular speed is slower, so it seems appropriate as compensation. In climb (J=0.45), α goes up to 13°, what may be even possible without stall, but is still merely 5.5° at the tip. Only in the static case (J=0.00), α goes from 11° to 29° so serious blade stall has to be expected.

This is only a simple estimate since equal (average) slipstream speed has been assumed over the whole propeller disk and the swirl has been entirely neglected. But it could come close to reality in the climb and cruise cases and even really close in the "pitch speed" case: no slipstream overall, no positive or negative local angles-of-attack either (except in the not relevant inner 30%), and no swirl.

If nothing else, this illustrates how and why the APC 17x12 E is a "better" propeller than the "cheap" Günther toy propeller.

Momentum Theory

At his Web page "How a Propeller Works", Martin Hepperle sketches a "stream tube passing through a propeller". The stream is accelerated by the propeller what makes for repulsive thrust T. It is faster than the surrounding air stream and hence also called slipstream. It is contracted while it is accelerated (because pressure is virtually constant), half of the acceleration and contraction taking place in front of the propeller and half behind. So, if the difference to the surrounding airspeed v far behind the propeller is Δv, the difference when passing through the propeller disk is $\Delta v/2$. The conservation-of-momentum law allows to draw an equation for thrust:

$$T = \frac{\pi}{4} D^2 \cdot \left(v + \frac{\Delta v}{2} \right) \cdot \rho \cdot \Delta v = \dot{m} \cdot \Delta v$$

The first term is the propeller disk area, the second is the speed of the slipstream passing through the disk, so their product is the volume flow through the propeller. That multiplied by air density is the mass flow $\dot{m}$ which, multiplied by the speed difference, results in the thrust force. This in turn is actually calculated from the thrust coefficient:

$$T = c_T \cdot \rho \cdot n^2 \cdot D^4$$

We equate both formulas, leaving out the terms equal on both sides:

$$\frac{\pi}{4} \cdot \left(v + \frac{\Delta v}{2} \right) \cdot \Delta v = c_T \cdot n^2 \cdot D^2$$

Expanding, replacing v by $J \cdot n \cdot D$, rearranging, and multiplying by 2 results in

$$\Delta v^2 + 2 \cdot J \cdot n \cdot D \cdot \Delta v - \frac{8 \cdot c_T \cdot n^2 \cdot D^2}{\pi} = 0$$ – another quadratic formula, with

$$k_1 = 2 \cdot J \cdot n \cdot D \quad \text{and} \quad k_2 = -\frac{8 \cdot c_T \cdot n^2 \cdot D^2}{\pi}$$ as its combined constants.

The discriminant is always negative since we consider only positive variable values:

$$\Delta = k_2 - \frac{k_1^2}{4} = -\left(\frac{8 \cdot c_T}{\pi} + J^2 \right) \cdot n^2 \cdot D^2 < 0$$

So there are two possible solutions:

$$\Delta v_{1,2} = -\frac{k_1}{2} \pm \sqrt{\frac{k_1^2}{4} - k_2}$$

Δv_1 (with positive square root) is the correct solution since Δv_2 would be negative:

$$\Delta v = \left(\sqrt{J^2 + \frac{8 \cdot c_T}{\pi}} - J \right) \cdot n \cdot D$$ appears like a modification of $v = J \cdot n \cdot D$

Under the square root, the advance ratio J is complemented by a term essentially being the thrust coefficient c_T. Subtracting J leaves only this complement to J, which is multiplied by $n \cdot D$ to give the speed added in the slipstream, just like J is multiplied by $n \cdot D$ to give the surrounding airspeed. So the term in parentheses is actually an added advance ratio ΔJ, half of which is added at the propeller disk. Exactly like a wing has an additional induced angle-of-attack when it produces lift, the propeller's advance ratio is increased by an "induced advance-ratio" when it produces thrust.

The power needed to accelerate the slipstream is unavoidably dissipated. The usual equation involves mass flow and speed difference, but replacing the product of both by thrust T (first equation on previous page) gives a more illustrative equation:

$$P_{diss} = \frac{\dot{m}}{2} \cdot \Delta v^2 = T \cdot \frac{\Delta v}{2}$$

That means the propeller dissipates power proportional to the thrust it produces and to the speed added to the slipstream when it passes the propeller disk. That in turn corresponds to the equation for thrust power, which is the propeller's actual business:

$$P_{thrust} = T \cdot v$$

Propulsion efficiency η^* (aptly named for a propeller) is hence defined as the ratio of thrust power (effective power) and thrust power plus dissipated power:

$$\eta^* = \frac{P_{thrust}}{P_{thrust} + P_{diss}} = \frac{T \cdot v}{T \cdot v + T \cdot \frac{\Delta v}{2}} = \frac{v}{v + \frac{\Delta v}{2}}$$

This is the classic explanation for the fact that thrust power is more efficiently produced by slightly accelerating a big propeller's slipstream than by strongly accelerating a small one's. Especially at low airspeeds v, the speed added to the slipstream Δv has to be low as well to keep propulsion efficiency η^* at a reasonable level.

Replacing v by $J \cdot n \cdot D$ and Δv by the equation drawn above yields:

$$\eta^* = \frac{J}{J + \frac{\Delta J}{2}} = \frac{2 \cdot J}{J + \sqrt{J^2 + \frac{8 \cdot c_T}{\pi}}} = \frac{2}{1 + \sqrt{1 + \frac{8 \cdot c_T}{\pi \cdot J^2}}}$$

That allows to calculate propulsion efficiencies η^* from measured or calculated data, and in fact that has been done for the diagrams shown in the Coefficient Comparison and Efficiency sections.

The third form may seem more elegant than the second but there will be a division by zero if advance ratio J is zero. This can be avoided by setting propulsion efficiency η^* to zero for zero advance ratio J and using the third form of the equation for advance ratios J greater than zero. Anyway, the values are higher than those calculated by JavaProp, but by 1.2% at most in the curve's middle part.

The here so-called "induced advance-ratio" has been illustratively used (to calculate local "angles-of-advance" φ) in the Pitch Distribution section:

$$J_i = \frac{\Delta J}{2} = \frac{1}{2}\left(\sqrt{J^2 + \frac{8 \cdot c_T}{\pi}} - J\right)$$

While the propeller is traveling at advance ratio J with respect to the free air-stream, it is actually in the slipstream, which is by $\Delta v/2$ faster than the surrounding free air-stream, what is expressed by the induced advance ratio J_i. The propeller is "slipping" in the air-stream, what is called slippage, and hence also the term slipstream. The propeller blades see an angle-of-attack smaller than that corresponding to the free-air-stream advance ratio J due to half of the added slipstream speed, represented by the induced advance ratio J_i. That gives another definition of propulsion efficiency:

$$\eta^* = \frac{J}{J + J_i}$$

Drive Illustration

Applied Solution

Combining a motor-gear combination and a propeller yields a drive by this chapter's word usage. The equation for the drive's rotational speed n, derived in the Applicable Solution section, is now applied to every advance ratio J present in the results of a propeller calculation or measurement, respectively. The polynomial constants K_1, K_2, and K_3 are calculated beforehand using the motor, gear, and propeller constants given in the case at hand. ("Motor" includes battery and ESC, just to put it simply.)

So every – calculated or measured – propeller power coefficient c_P results in a certain rotational speed n, which in turn is converted into flight speed v using the respective advance ratio J. Finally, all interesting figures are calculated for every resulting flight speed so they can be plotted in diagrams over the drive's whole flight speed range. That means flight speeds from zero ("static") to "pitch speed" (zero thrust) because in most if not all cases no negative-thrust coefficients (windmilling propeller) are calculated or measured.

The formulas have been taken from the sections Applicable Solution, Mechanical-Aerodynamic Conversion, Mechanical-Mechanical Conversion, and Characteristic Speeds and Quantities, and they are applied in the following order:

$$n = \frac{K_2 + \sqrt{K_2^2 + c_P K_3 K_1}}{c_P K_3} \quad [s^{-1}] \qquad v = J \cdot n \cdot D \quad [m/s]$$

$$T = c_T \cdot \rho \cdot n^2 \cdot D^4 \quad [N] \qquad P_{thrust} = T \cdot v \quad [W]$$

$$P_{mech} = c_P \cdot \rho \cdot n^3 \cdot D^5 \quad [W] \qquad M_g = \frac{P_{mech}}{2 \cdot \pi \cdot n} \cdot 100 \quad [Ncm]$$

$$I = \left(U_b - \frac{i_g}{k_V} \cdot n \cdot 60 \right) \cdot \frac{1}{R} \quad [A] \qquad P_{el} = U_b \cdot I \quad [W]$$

$$\eta_p = \frac{P_{thrust}}{P_{mech}} \cdot 100 \quad [\%] \qquad \eta_d = \frac{P_{mech}}{P_{el}} \cdot 100 \quad [\%] \qquad \eta = \frac{P_{thrust}}{P_{el}} \cdot 100 \quad [\%]$$

Rotational speed n [s^{-1}] is rotations *per second* here because that is required for propeller calculations. Hence M_g needs no multiplication by 60 but it is multiplied by 100 to have convenient numbers in [Ncm] instead of [Nm]. Of course, there may be additional columns with n [min^{-1}], v [km/h], or v [mph] for convenience as well. Current (amperage) I needs the multiplication by 60 here. Efficiencies are multiplied by 100 to have convenient two-digit numbers in [%].

The calculations are performed twice, for full-power and for cruise-power setting.

The full-power case is meant for *climb*. The following table is calculated for our example drive (motor/gear and propeller) at its nominal 8.4 V battery voltage U_b. Additional columns as well as efficiencies are left out for lack of page width.

J	c_P	c_T	n	v	T	P_{thrust}	P_{mech}	M	I	P_{el}
[-]	[-]	[-]	[1/min]	[m/s]	[N]	[W]	[W]	[Ncm]	[A]	[W]
0,00	0,12445	0,13799	6804	0,0	2,04	0,0	36,5	5,13	8,5	71,7
0,05	0,08813	0,12009	7413	1,1	2,11	2,3	33,4	4,31	7,3	61,2
0,10	0,10927	0,14835	7037	2,1	2,35	4,8	35,5	4,81	8,1	67,7
0,15	0,11751	0,15737	6907	3,0	2,40	7,2	36,1	4,99	8,3	69,9
0,20	0,12102	0,15838	6854	4,0	2,38	9,5	36,3	5,06	8,4	70,8
0,25	0,12248	0,15489	6833	5,0	2,31	11,5	36,4	5,09	8,5	71,2
0,30	0,12170	0,14826	6844	6,0	2,22	13,3	36,4	5,07	8,5	71,0
0,35	0,11894	0,14045	6886	7,0	2,13	14,9	36,2	5,02	8,4	70,3
0,40	0,10057	0,12027	7183	8,4	1,98	16,6	34,7	4,62	7,8	65,2
0,45	0,09208	0,10832	7337	9,6	1,86	17,9	33,9	4,41	7,4	62,5
0,50	0,08585	0,09705	7457	10,9	1,72	18,7	33,2	4,25	7,2	60,4
0,55	0,07852	0,08472	7609	12,2	1,57	19,1	32,2	4,04	6,9	57,8
0,60	0,07029	0,07188	7792	13,6	1,39	19,0	31,0	3,80	6,5	54,6
0,65	0,06112	0,05839	8017	15,2	1,20	18,2	29,3	3,49	6,0	50,8
0,70	0,05088	0,04423	8298	16,9	0,97	16,5	27,1	3,12	5,5	45,9
0,71	0,04867	0,04128	8364	17,3	0,92	16,0	26,5	3,03	5,3	44,8
0,72	0,04643	0,03830	8432	17,7	0,87	15,4	25,9	2,94	5,2	43,6
0,73	0,04420	0,03524	8502	18,1	0,81	14,7	25,3	2,84	5,0	42,4
0,74	0,04189	0,03225	8577	18,5	0,76	14,0	24,6	2,74	4,9	41,1
0,75	0,03955	0,02924	8656	18,9	0,70	13,2	23,9	2,64	4,7	39,7
0,76	0,03711	0,02614	8740	19,4	0,64	12,4	23,1	2,52	4,6	38,3
0,77	0,03462	0,02300	8830	19,8	0,57	11,4	22,2	2,40	4,4	36,7
0,78	0,03212	0,01988	8924	20,3	0,51	10,3	21,3	2,28	4,2	35,1
0,79	0,02959	0,01676	9022	20,8	0,44	9,1	20,2	2,14	4,0	33,4
0,80	0,02709	0,01346	9124	21,3	0,36	7,6	19,2	2,01	3,8	31,6
0,81	0,02441	0,01021	9238	21,8	0,28	6,1	17,9	1,85	3,5	29,7
0,82	0,02167	0,00692	9360	22,4	0,19	4,3	16,6	1,69	3,3	27,6
0,83	0,01894	0,00366	9488	23,0	0,11	2,4	15,1	1,52	3,0	25,4
0,84	0,01616	0,00038	9626	23,6	0,01	0,3	13,4	1,33	2,7	23,0
0,85	0,01327	-0,00302	9777	24,2	-0,09	-2,2	11,6	1,13	2,4	20,4

The first three columns are just copied from the JavaProp propeller calculation results. The tool increased the advance ratio J in 0.05 steps until 0.70 and then in small 0.01 steps until at 0.85 the thrust coefficient c_T got smaller than zero (windmilling propeller). The following column shows the propeller's rotational speed n [min^{-1}] as rotations *per minute* since this is the figure shown in diagrams.

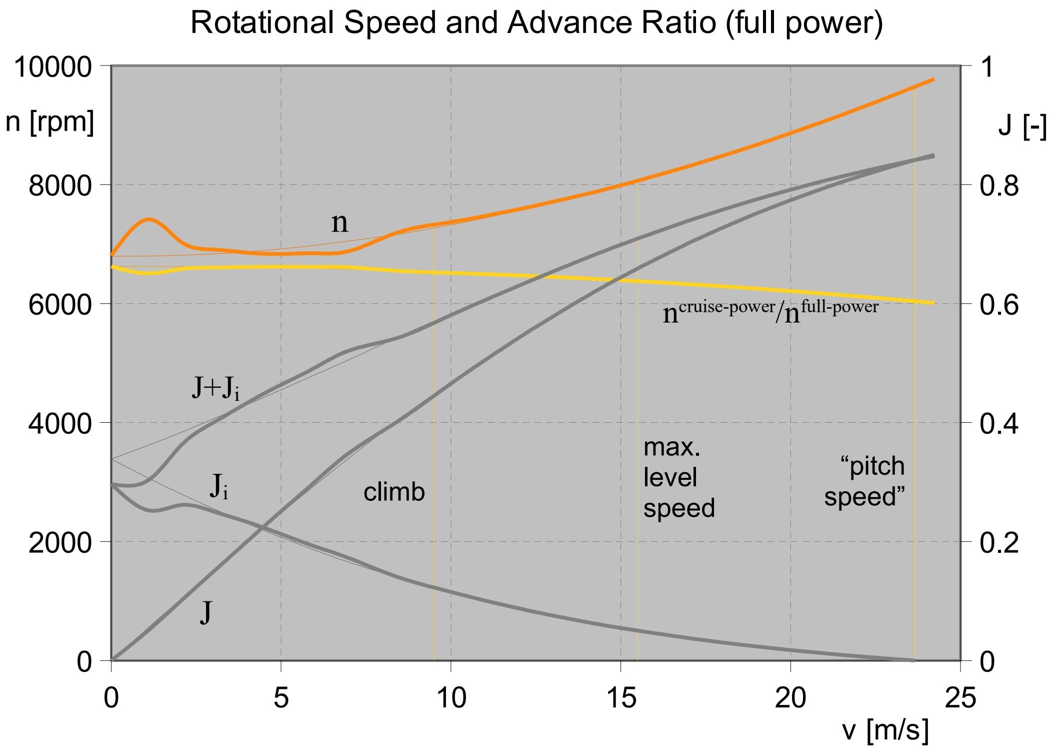
The following *cruise*-power table is calculated assuming the ESC is set to an equivalent 0.6 voltage reduction factor, giving 5.0 V equivalent battery voltage U_b. The required factor is known from the performance calculations.

J	c_P	c_T	n	v	T	P_{thrust}	P_{mech}	M	I	P_{el}
[-]	[-]	[-]	[1/min]	[m/s]	[N]	[W]	[W]	[Ncm]	[A]	[W]
0,00	0,12445	0,13799	4507	0,0	0,90	0,0	10,6	2,2	4,1	20,7
0,05	0,08813	0,12009	4823	0,7	0,89	0,6	9,2	1,8	3,5	17,4
0,10	0,10927	0,14835	4629	1,4	1,02	1,4	10,1	2,1	3,9	19,4
0,15	0,11751	0,15737	4561	2,0	1,05	2,1	10,4	2,2	4,0	20,1
0,20	0,12102	0,15838	4533	2,6	1,04	2,7	10,5	2,2	4,1	20,4
0,25	0,12248	0,15489	4522	3,3	1,01	3,3	10,5	2,2	4,1	20,5
0,30	0,12170	0,14826	4528	4,0	0,97	3,8	10,5	2,2	4,1	20,5
0,35	0,11894	0,14045	4550	4,6	0,93	4,3	10,4	2,2	4,0	20,2
0,40	0,10057	0,12027	4705	5,5	0,85	4,7	9,8	2,0	3,7	18,6
0,45	0,09208	0,10832	4784	6,3	0,79	5,0	9,4	1,9	3,6	17,8
0,50	0,08585	0,09705	4845	7,1	0,73	5,1	9,1	1,8	3,4	17,2
0,55	0,07852	0,08472	4921	7,9	0,66	5,2	8,7	1,7	3,3	16,4
0,60	0,07029	0,07188	5011	8,8	0,58	5,1	8,2	1,6	3,1	15,5
0,65	0,06112	0,05839	5119	9,7	0,49	4,7	7,6	1,4	2,9	14,4
0,70	0,05088	0,04423	5251	10,7	0,39	4,2	6,9	1,2	2,6	13,0
0,71	0,04867	0,04128	5281	10,9	0,37	4,0	6,7	1,2	2,5	12,7
0,72	0,04643	0,03830	5312	11,2	0,35	3,9	6,5	1,2	2,5	12,4
0,73	0,04420	0,03524	5344	11,4	0,32	3,7	6,3	1,1	2,4	12,1
0,74	0,04189	0,03225	5377	11,6	0,30	3,5	6,1	1,1	2,3	11,7
0,75	0,03955	0,02924	5412	11,8	0,27	3,2	5,8	1,0	2,3	11,4
0,76	0,03711	0,02614	5450	12,1	0,25	3,0	5,6	1,0	2,2	11,0
0,77	0,03462	0,02300	5489	12,3	0,22	2,7	5,3	0,9	2,1	10,6
0,78	0,03212	0,01988	5529	12,6	0,19	2,4	5,1	0,9	2,0	10,2
0,79	0,02959	0,01676	5571	12,8	0,17	2,1	4,8	0,8	1,9	9,7
0,80	0,02709	0,01346	5614	13,1	0,14	1,8	4,5	0,8	1,9	9,3
0,81	0,02441	0,01021	5661	13,4	0,10	1,4	4,1	0,7	1,8	8,8
0,82	0,02167	0,00692	5711	13,7	0,07	1,0	3,8	0,6	1,7	8,3
0,83	0,01894	0,00366	5763	14,0	0,04	0,5	3,4	0,6	1,6	7,8
0,84	0,01616	0,00038	5817	14,3	0,00	0,1	3,0	0,5	1,4	7,2
0,85	0,01327	-0,00302	5876	14,6	-0,03	-0,5	2,5	0,4	1,3	6,6

The first three columns are exactly like in the previous table since they have been calculated in JavaProp for one rotational speed only (see previous chapter). The following columns are different (lower values) due to the reduced battery voltage U_b. Like in the previous table, all calculated values are displayed with few digits but have far more precision internally (in the spreadsheet).

Rotational Speed

The pivotal variable in our solution is the propeller's (or gear output shaft's) *rotational* speed n (see chapter Basic Solution). Eventually the drive's behavior can be described over a whole *flight* speed v range from "static" (zero speed) to "pitch speed" (zero thrust). So rotational speed n is the first variable to consider over this flight speed v range for our example drive, particularly in the full-power case:



The drive's behavior is mainly determined by the propeller. The example propeller's coefficient curves had been smoothened by applying second-order (quadratic) polynomials. Now every drive variable has been calculated twice, from the original and from the smoothened coefficient curves. In the diagrams, the smoothened curves are thinner and darker than the original ones. They give a clue if the warps in the lower speed range (below about 12 m/s) actually have any effect.

While this is useful for an example, coefficient curves which are inverted parabolas are *not typical* but a peculiarity of the "cheap" example propeller (see chapter Propeller Illustration). Hence all curves in the following diagrams are not typical as well in terms of their shape. The "better" propellers considered by way of comparison will be considered again later in this chapter to illustrate the differences.

The basic course of the curves *is typical*, though. Rotational speed n increases progressively with increasing flight speed v. For comparison, the propeller's advance ratio J and the induced advance ratio J_i as well as their sum $J+J_i$ have been included in the diagram. The latter looks like a mirror of the rotational speed curve, having the same warps in the opposite direction and opposite curvature.

The induced advance ratio J_i is an abstract measure of the thrust produced by the propeller. From the static maximum (zero flight speed), it *de*creases degressively. It stands to reason that rotational speed increases with less thrust, meaning less torque needed to spin the propeller. As a result, the advance ratio J *in*creases degressively.

The *amount* of rotational-speed increase – colloquially called "unloading" – is even an aspect of practical value in this consideration. Best climb speed and maximum level-flight speed are known from the performance calculations. Zero-thrust speed – colloquially called "pitch speed" – results from the drive calculations. These three speeds have been highlighted in the diagram by vertical lines.

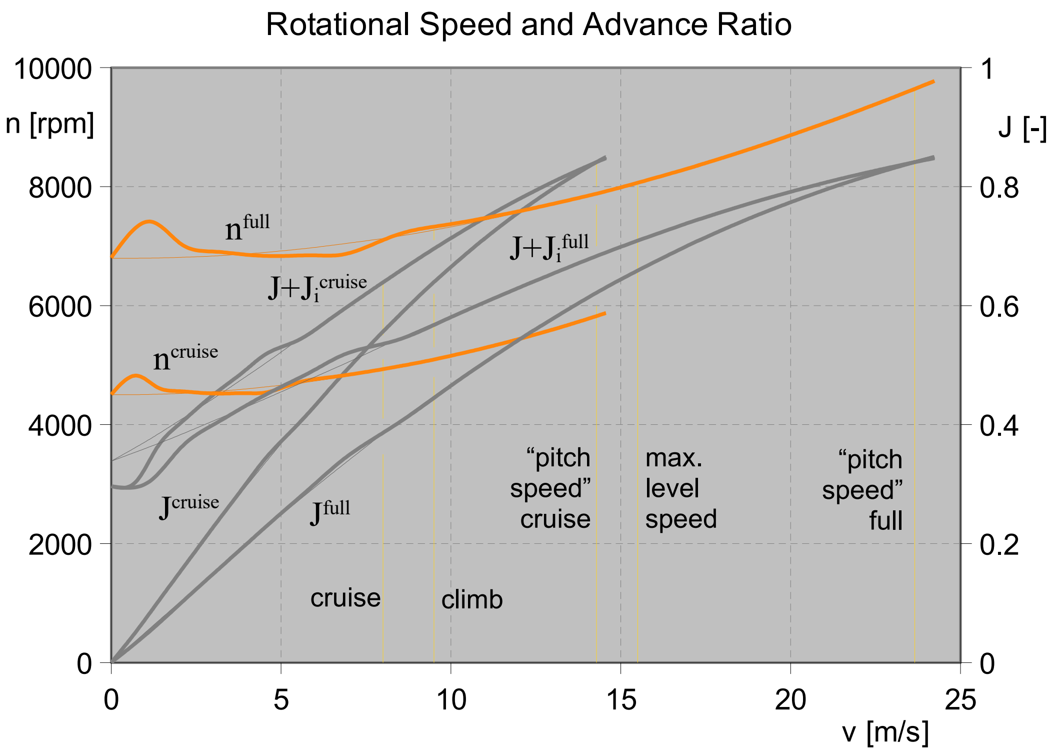
From "static" (zero flight speed), the propeller's rotational speed increases noticeably:

"static"	6800 rpm	100%
climb	7320 rpm	108%
max. level speed	8065 rpm	119%
"pitch speed"	9615 rpm	141%

However, "pitch speed" has no practical value because it can be reached only in a steep dive. At the barely practical maximum level-flight speed, the "unload" is only 19%. And at best climb speed, which is actually flown at full power, the "unload" is not more than 8%. That is even quite much.

From the Motor/Gear Illustration chapter (Basic Characteristics section) we know that a small, high-k_V, and "cheap" drive (like this example drive) is more *elastic* than a big, low-k_V, and "better" drive. That means its rotational speed n increases more with the same increase of flight speed v (and its advance ratio J increases more degressively).

When the drive is set to less than full power, all that should be basically the same. To see how the equivalent (to power setting) battery voltage U_b affects ("scales") the curves, the cruise-power case is added to the diagram:



The induced advance ratio J_i is omitted and cruise speed and "pitch-speed" markings are added. Obviously, all curves are scaled horizontally (on the flight-speed axis) by 0.604 ("pitch-speed" ratio). That is – not incidentally – only slightly more than the 0.6 (exactly 0.595) voltage reduction factor (equivalent to 60% power setting).

Vertically, the rotational speed n curve is scaled by 0.604 (same as horizontally) at the respective "pitch speed" but by 0.662 at zero flight speed ("static"). The explanation is not obvious: In the equation for rotational speed n, full power and cruise power are represented by the respective equivalent battery voltage U_b in the polynomial constant K_1 (stall torque) only. It is in the same term like the polynomial constant K_3 (propeller torque) and the propeller's power coefficient c_P.

Shaft power is not proportional to rotational speed so in the "static" case the latter is relatively higher at cruise power than at full power. At "pitch speed", the propeller's power coefficient c_P is only 13% of the "static" value so the influence of propeller power on rotational speed is smaller and the influence of K_1 and hence U_b prevails. The formulas for the two cases are meant to illustrate this:

$$\frac{n^{cruise,static}}{n^{full,static}} = \frac{K_2 + \sqrt{K_2^2 + c_P^{static} K_3 K_1^{cruise}}}{K_2 + \sqrt{K_2^2 + c_P^{static} K_3 K_1^{full}}} = 0.662$$

$$\frac{n^{cruise,pitch}}{n^{full,pitch}} = \frac{K_2 + \sqrt{K_2^2 + c_P^{pitch} K_3 K_1^{cruise}}}{K_2 + \sqrt{K_2^2 + c_P^{pitch} K_3 K_1^{full}}} = 0.604$$

Horizontal and vertical scaling are equal at the "pitch speed" point – and any other point (advance ratio J) as well – for a simple reason: $v = J \cdot n \cdot D$ – and J as well as D are the same or constant, respectively. The scaling factor n^{cruise}/n^{full} over the whole range is shown as a curve in the first diagram in this section. Like the $J{+}J_i$ curve, it looks like a mirror of the rotational speed n curve. It makes for less "unloading" in the cruise-power case than in the full-power case.

Anyway, the advance-ratio curves are not scaled vertically because advance ratios are dimensionless and because the same set of propeller coefficients has been used for both the full-power and the cruise-power cases. The latter is also the reason why there are the same coefficients in the numerator and the denominator of the rotational speed n scaling ratios (for instance the two formulas above).

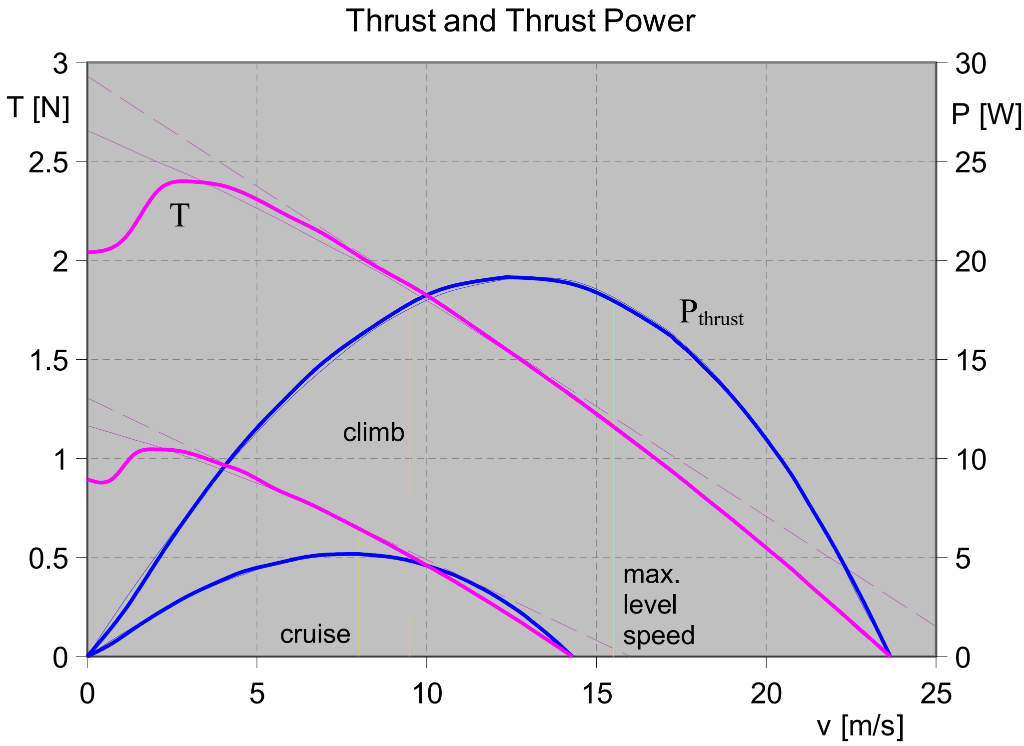
Derived Variables

The same style of diagram like in the previous section is used in this section for variables derived from the basic solution: thrust, moment (torque), current (amperage), and the related powers as well as efficiencies. This follows the order of derivation shown by the respective formulas in the Applied Solution section above.

There are the respective curves for the full-power and cruise-power cases. There are vertical speed markings for cruise (8 m/s), climb (9.5 m/s), maximum level speed (15.5 m/s), as well as cruise-power and full-power "pitch speed" (14.3 / 23.7 m/s).

Thrust, moment (torque), and current (amperage) are shown together with their related powers in one diagram each. This way, possibly different scaling can be compared, that is vertical scaling. Obviously, horizontal scaling is always the same as what we have seen in the previous section: approximately by the 0.6 voltage reduction factor, or 60% power setting.

All three powers are repeated in an own diagram as well as the three efficiencies are repeated in an own, last diagram because they are ratios of these powers. So five diagrams show all derived variables, just for the sake of completeness.



Thrust T is proportional to rotational speed squared n^2, which increases with flight speed v, and the thrust coefficient c_T, which decreases. In any case, thrust decreases but it may decrease progressively – like in this case – or linearly or even degressively.

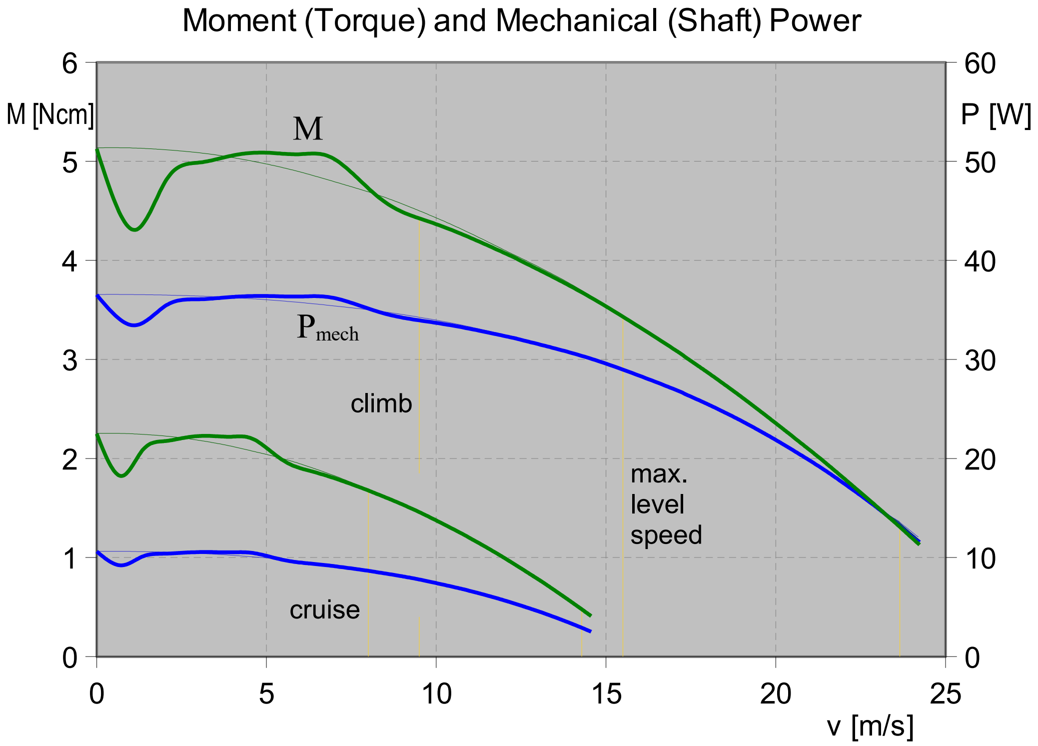
As mentioned in the Propeller Illustration chapter (Propeller Diagram section), we know that static thrust really is like calculated and not like the smoothened curve suggests. How the curve really runs between zero and about 9 m/s flight speed (or 6 m/s at cruise power) is not known, though, so the calculated curve is *not typical*. Typical would be kind of an inverted parabola.

More important, the second part above 9 m/s (or 6 m/s) would be typically a straight line. This is the case if the propeller coefficients are calculated with JavaProp and the propeller is a "better" one without blade stall in the higher speed range. However, this "cheap" propeller is without blade stall only between about 9 m/s and 11 m/s at full power or 6.5 m/s and 7 m/s at cruise power. The dashed lines show how thrust might run linearly if there would be no blade stall. Cruise and climb speed are in or close to the respective stall-free speed ranges.

Thrust power P_{thrust} is thrust times flight speed, hence it is zero at zero flight speed ("static") and at zero thrust ("pitch speed"). The curve is an inverted parabola, somewhat skewed because thrust runs non-linear. The warps in the thrust coefficient c_T curve are hardly transferred to the thrust-power curve because they are in the low flight-speed range and hence marginalized by the thrust-times-speed multiplication.

In the full-power case, best climb speed (9.5 m/s) is lower than the speed of maximum thrust-power (12.5 m/s) – typical for model airplanes. Cruise speed (8 m/s) being equal to or slightly higher than the speed of maximum thrust-power in the cruise-power case is typical as well. And maximum level speed (15.5 m/s) is always higher than the speed of maximum thrust-power (12.5 m/s) in the full-power case.

Static thrust is scaled down from full-power to cruise-power by factor 0.439 what is 0.662 squared. That is because static rotational speed is scaled by factor 0.662 and thrust is proportional to rotational speed squared.



Moment M (torque) is – as well as thrust T – proportional to rotational speed squared n^2, which increases with flight speed v, and the power coefficient c_P, which decreases. In any case, torque decreases progressively.

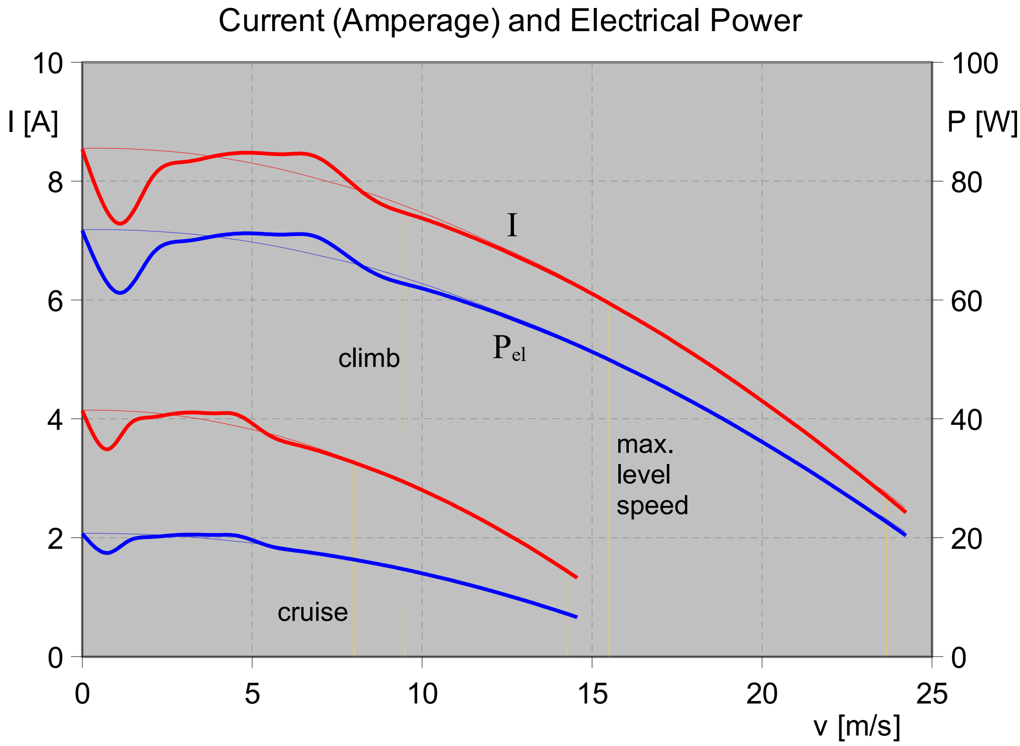
From the Propeller Illustration chapter (Propeller Diagram section) we know that static torque really is like calculated *and* like the smoothened curve suggests. How the curve really runs between zero and about 12 m/s flight speed is still not known, though. It may be like the smoothened curve or there may be some smaller warps than in the calculated curve.

Mechanical power P_{mech} is also called shaft power and is torque times rotational speed (and 2π) so it is proportional to rotational speed cubed n^3, which increases with flight speed v. That makes the warps in the shaft-power curve smaller than in the torque curve. Shaft power always decreases progressively.

Both torque M and shaft power P_{mech} are not zero at "pitch speed" (zero thrust) because the propeller still needs a noticeable amount of torque to spin at the quite high rotational speed there.

"Static", torque is scaled down from full-power to cruise-power by factor 0.439 what is 0.662 squared, and rotational speed is scaled by factor 0.662. At "pitch speed", torque is scaled down by factor 0.365 what is 0.604 squared, and rotational speed is scaled by factor 0.604. Torque is proportional to rotational speed squared.

"Static", shaft power is scaled down from full-power to cruise-power by factor 0.291 what is 0.662 cubed, and rotational speed is scaled by factor 0.662. At "pitch speed", shaft power is scaled down by factor 0.220 what is 0.604 cubed, and rotational speed is scaled by factor 0.604. Shaft power is proportional to rotational speed cubed.



Current I (amperage) is – other than thrust T and torque M – inversely proportional to rotational speed n, which increases with flight speed v. Still, amperage decreases progressively in the same way torque does. The warps in the power coefficient c_P curve are transferred to the amperage curve via rotational speed n.

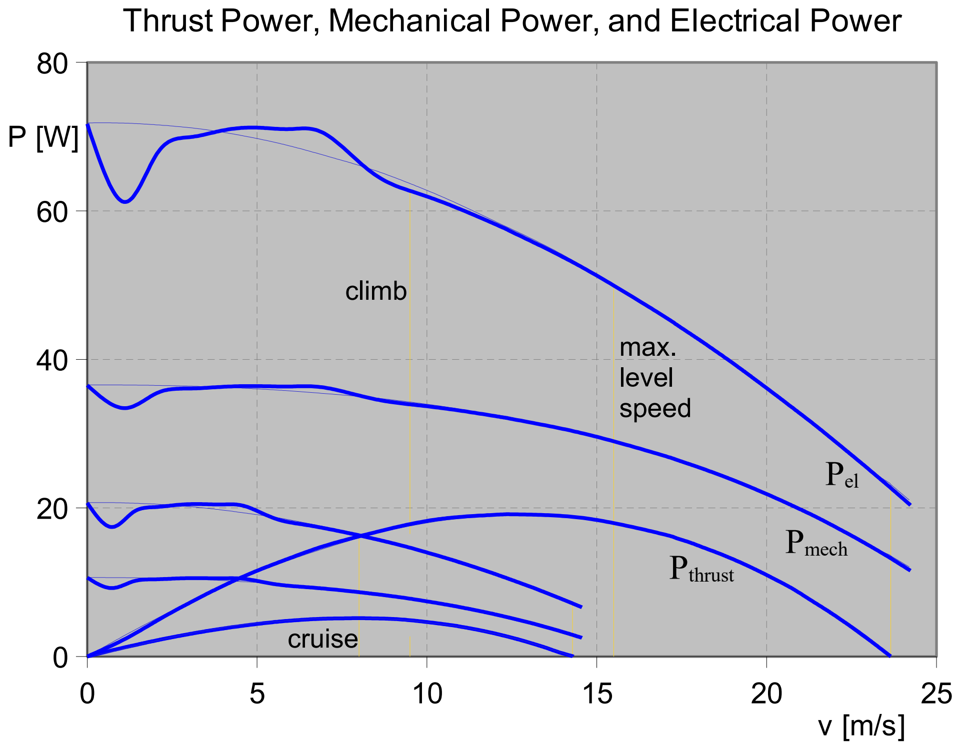
Electrical power P_{el} is simply amperage times (equivalent) battery voltage, which is constant. Hence the curve's shape is the same as the amperage curve's – the warps as well as the progressive decrease.

Both amperage I and electrical power P_{el} are not zero at "pitch speed" (zero thrust) for the same reason as torque M and shaft power P_{mech}: because the propeller still needs a noticeable amount of torque – and hence amperage – to spin at the quite high rotational speed there.

"Static", amperage is scaled down from full-power to cruise-power by factor 0.485 while rotational speed is scaled by factor 0.662. At "pitch speed", amperage is scaled down by factor 0.529 while rotational speed is scaled by factor 0.604. Amperage is *inversely* proportional to rotational speed so its "pitch speed" scaling factor is higher than its "static" scaling factor. Equivalent battery voltage U_b is a constant term not only in the rotational speed formula but in the amperage formula as well. So it acts twice in the scaling factors, as it were, and makes them smaller here.

"Static", electrical power is scaled down from full-power to cruise-power by factor 0.288, while rotational speed is scaled by factor 0.662. At "pitch speed", electrical power is scaled down by factor 0.315, while rotational speed is scaled by factor 0.604. Since electrical power is amperage times equivalent battery voltage, its scaling factors are the respective amperage scaling factors times the 0.595 voltage reduction factor.

Static amperage and voltage, so power as well, could be (and have been) measured. This calculation predicts 8.5 A and 72 W. The first value is useful for selecting an ESC and the second, colloquially called "power in", is used as an approximate indication whether the drive is powerful enough for the airplane.



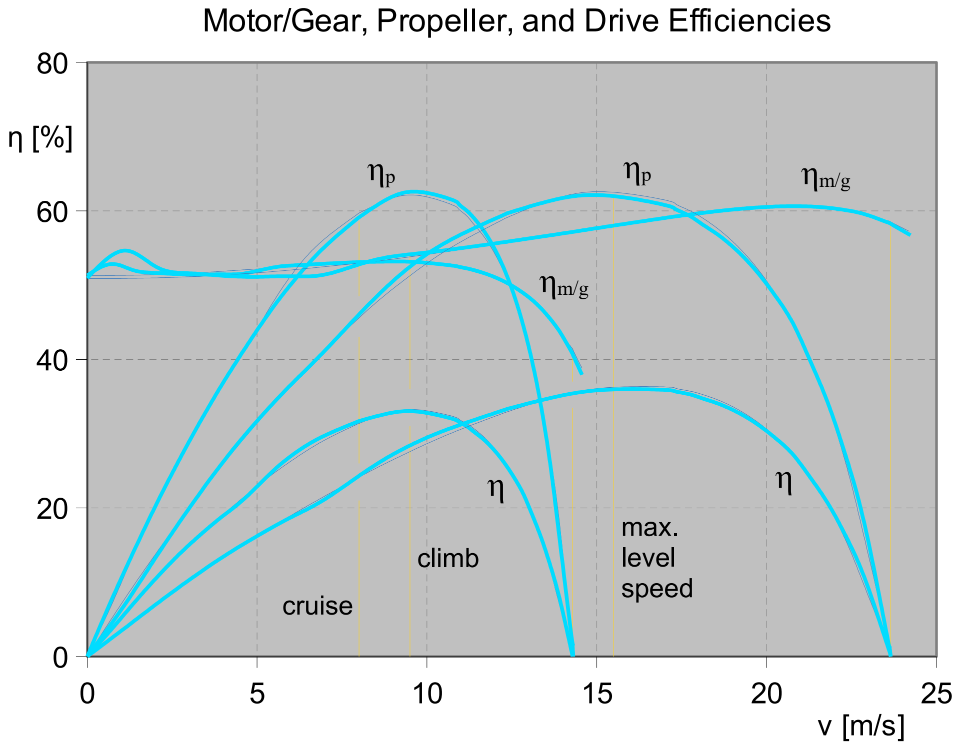
Electrical, mechanical, and thrust power together illustrate the losses of power from the battery to the propeller, as it were. Which *portion* of the respective initial power is *not* lost, though, is hard to tell from this diagram so these ratios, called efficiencies, are explicitly shown in the next diagram.

Here, it is worth noting that the usable full-power speed range from best climb speed to maximum level speed is the range of maximum thrust-power. In this range, mechanical as well as electrical power decrease when speed increases hence *all efficiencies* increase. This is typical for the full-power case and actually intended to have maximum thrust power even accepting less-than-maximum efficiency.

Then again, cruise speed is about equal to the speed of maximum thrust-power at cruise-power. Because thrust power decreases at lower and higher speeds, *total efficiency* is definitely lower at lower speeds and tends to be lower at higher speeds. This is typical for the cruise-power case. This is again intentional design or at least a happy chance, respectively.

Cruise power has been *chosen* so the airplane flies as slow – and economically – as possible but still has some speed stability. More cruise power would scale the power curves to higher power as well as higher speed so that cruise speed will still *roughly* coincide with maximum thrust-power speed. Put another way, cruise speed will *roughly* coincide with maximum thrust-power speed *in level flight*, regardless of power setting (even up to full-power). Hence it is typical for level flight that total efficiency is close to its maximum.

The warps in the power curves stem from the propeller's power-coefficient curve. This diagram shows that these warps are not relevant: They occur only in the speed range below the usable speed range and the calculated power values are even correct "statically", at zero flight speed. Taken with a grain of salt, this can be seen as typical.



The motor/gear combination's efficiency $\eta_{m/g}$ curve over the whole *rotational speed* n range is an inverted parabola skewed to the right. Over the whole *flight speed* v range, rotational speed increases progressively from 64% of idle speed "static" to 76% at "pitch speed". This cuts a small part (12%) of the original efficiency curve including the maximum and makes the cutout's left part progressively increasing.

The propeller's efficiency η_p curve over the whole *advance ratio* J range is an inverted parabola skewed to the right, too. Over the whole *flight-speed* v range, the advance ratio increases degressively from "static" to "pitch speed" so the whole original efficiency curve is somewhat skewed to the left, lessening its right-skew.

Total drive-efficiency η is the product of the two other efficiencies. So the curve is a scaled-down propeller efficiency curve with the scaling factor (motor/gear efficiency) not quite constant. At full-power, this factor increasing to the right skews the curve slightly to the right so maximum drive efficiency occurs at higher flight-speed than maximum propeller efficiency. At cruise-power, motor/gear efficiency is almost constant so propeller efficiency and drive efficiency are close to their respective maximum at about the same flight speed.

Both propeller-efficiency η_p curves (for cruise-power and full-power) are merely horizontally scaled but not vertically because no different coefficient curves have been (or even could have been) calculated (see Propeller Illustration chapter), hence they have the same maximum.

The two motor/gear-efficiency $\eta_{m/g}$ curves are vertically scaled by equivalent battery voltage. The curves seem to coincide in the low speed range, though, so it looks like this efficiency being dependent on flight speed but not on power setting *in level flight*.

Hence the total drive-efficiency η curves are slightly scaled vertically, maximum efficiency being slightly higher at full power (due to higher equivalent battery voltage). But at climb speed, efficiency is even lower than at cruise speed because the propeller is more loaded. This is typical for most of all drives.

Drive Comparison

While three motor/gear combinations and the three corresponding propellers have been separately compared in the previous two chapters, they are now compared together as complete drives. Basic specifications of models and drives are repeated here and the respective basic drive characteristics for the *full-power* case are added:

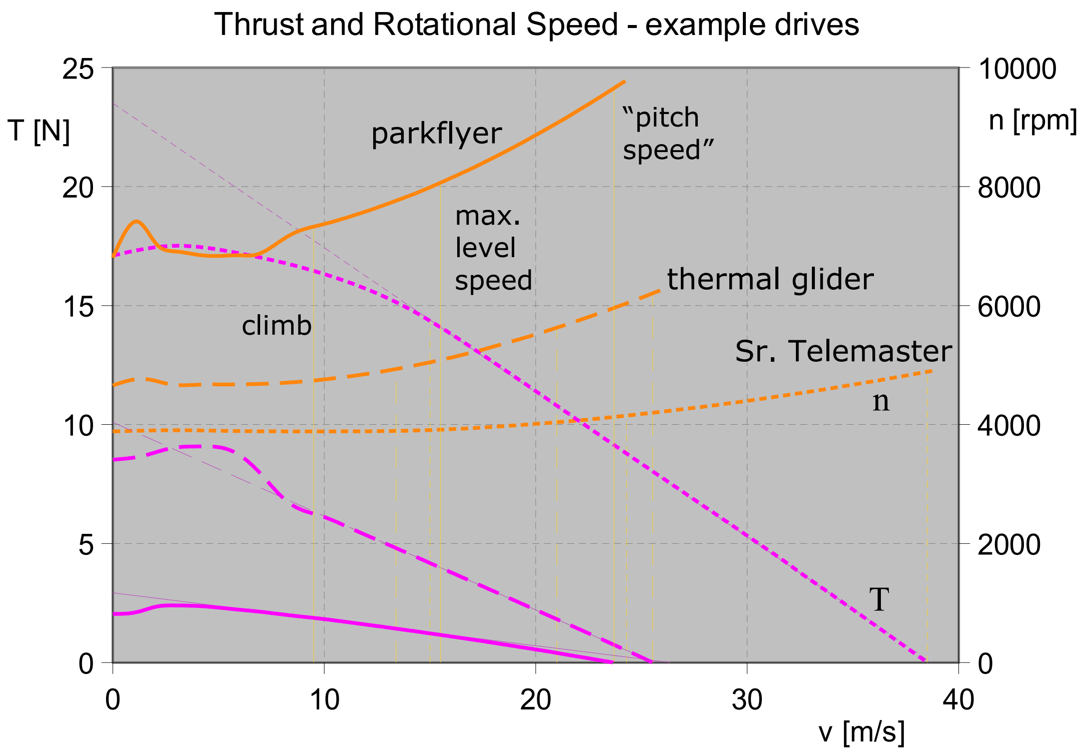
Type of model	55" retro parkflyer	100" thermal glider	95" Sr. Telemaster
Weight W	0.85 kg / 1.9 lbs	1.7 kg / 3.75 lbs	4.5 kg / 10 lbs
Wing loading	32 g/dm^2 / 11 oz/ft^2	38 g/dm^2 / 13 oz/ft^2	53 g/dm^2 / 18 oz/ft^2
Motor	400-size "can"	480-size premium	4130 brushless
Gear ratio i_g – η_g	2.3:1 – 89%	4.4:1 – 95%	no gear – 100%
Motor / Drive k_V	3000 / 1300 rpm/V	3440 / 780 rpm/V	360 / 360 rpm/V
Propeller	6.9x6.3" toy prop	CAM-Carbon 14x8	APC 17x12 E
Battery (voltage)	7s 1000 NiCd (8.4 V)	7s 2300 NiCd (8.4 V)	4s 5000 LiPo (14.8 V)
Static:			
power "in" P_{el}	70 W (8.3 A)	150 W (18 A)	500 W (35 A)
Power/Weight	82 W/kg / 37 W/lb	88 W/kg / 40 W/lb	111 W/kg / 50 W/lb
thrust T	2 N / 0.46 lbf	8.5 N / 1.9 lbf	17 N / 3.8 lbf
Thrust/Weight	0.24	0.51	0.38
Speeds:			
"pitch speed"	23.7 m/s / 100%	25.5 m/s / 100%	38.5 m/s / 100%
maximum level	15.5 m/s / 65%	21.0 m/s / 82%	24.3 m/s / 63%
climb	9.5 m/s / 40%	13.4 m/s / 53%	15.0 m/s / 39%
climb rate / ratio	1.4 m/s / 1:6.9	3.3 m/s / 1:3.6	3.8 m/s / 1:4.0
"Unloading":			
static	6800 rpm / 100%	4655 rpm / 100%	3885 rpm / 100%
climb	7320 rpm / 108%	4935 rpm / 106%	3910 rpm / 101%
max. level speed	8065 rpm / 119%	5630 rpm / 121%	4155 rpm / 107%
"pitch speed"	9640 rpm / 142%	6205 rpm / 133%	4885 rpm / 126%
Efficiencies:			
climb/cruise/max.	28% / 32% /36%	49% / 48% / 53%	43% / 50% / 51%
cruise power	60%	46%	48%

From the parkflyer over the thermal glider to the Sr. Telemaster, the models' size, weight, wing loading, and speed increase. Their respective drives are bigger, more powerful, and "better". The latter means their efficiencies are noticeably higher, as specified in the comparison sections of the two previous chapters.

There is also shown that overall efficiency is in a large part affected by the propeller's power loading, which is especially low on the thermal glider. Consequently, and also due to its moderate-pitch propeller, it has the highest thrust/weight ratio despite a rather low power/weight ratio. Maximum level speed and climb speed come rather close to "pitch speed", and the airplane does a steep climb. At maximum level speed, the drive "unloads" especially much because the airplane is sleek and gets fast. Efficiency is higher in climb than in cruise and is close to maximum, which is rather high – the drive, especially the propeller, is optimal for climb.

The parkflyer and the Sr. Telemaster have several things in common, despite their different sizes and weights. Both are – relative to their sizes – low-wing-loading, slow-flying, and draggy airplanes, and their drives are optimal for cruise. Their climb speeds and even their maximum level speeds are substantially lower than their "pitch speeds". The parkflyer's drive is just so "cheap" (inefficient) and weak (see its cruise power setting) that it noticeably "unloads", and the airplane manages merely a mild climb. Then again, the Sr. Telemaster's drive is "better" (rather efficient) and strong so it "unloads" little and the airplane does a rather steep climb.

For better illustration, some characteristics need to be shown in diagrams. For all three example drives, the propeller coefficients have been calculated with JavaProp. Curiously, the resulting thrust coefficient c_T curves are straight where no blade stall is predicted, that is at higher advance ratios J. In the drive calculations, this translates into a just as straight part of the thrust T curve where no blade stall occurs, that is at corresponding higher flight speeds v. This is a distinctive characteristic of electric drives. In the following diagram, the straight parts of the actual thrust curves are overlaid with thinner and darker straight lines. They show which values of static thrust and "pitch speed" could be expected if there were no blade stall at all:

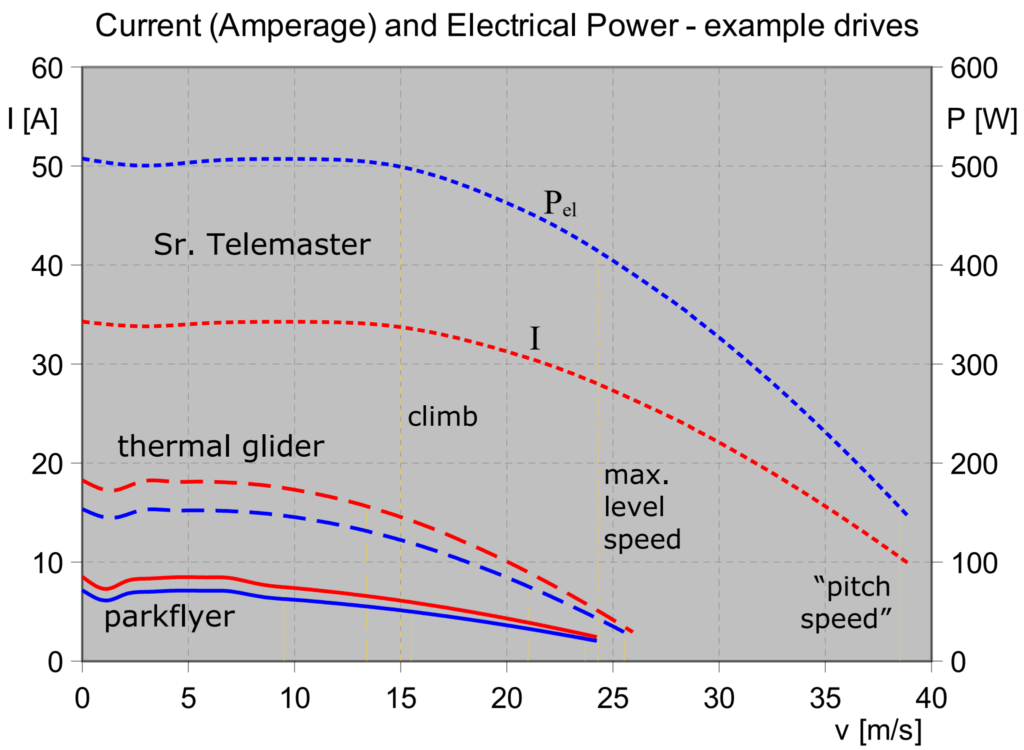


The "cheap" example drive (parkflyer) is without blade stall only between about 9 m/s and 11 m/s at full power, the "better" drives (thermal glider and Sr. Telemaster) above about 10 m/s or 15 m/s up to their "pitch speed", respectively.

The first and second examples (parkflyer, thermal glider) have nearly the same "pitch speed" but very different static thrust. The parkflyer drive is quite elastic, meaning flight speed decreases substantially if more thrust is needed. That is because it has a high specific rotational speed k_V (1300 rpm/V) making up for the propeller's small pitch (6.3 in) and because it is a small and "cheap" drive. The thermal-glider drive has a bit more pitch (8 in) but lower specific rotational speed k_V (780 rpm/V) and is bigger and "better" so it is less elastic, or more rigid. The Sr. Telemaster drive is "better" as well, has even more pitch (12 in) as well as higher battery voltage but an even lower specific rotational speed k_V (360 rpm/V) so it is even more rigid. It is much bigger and more powerful so it goes to substantially higher thrust and higher flight speeds.

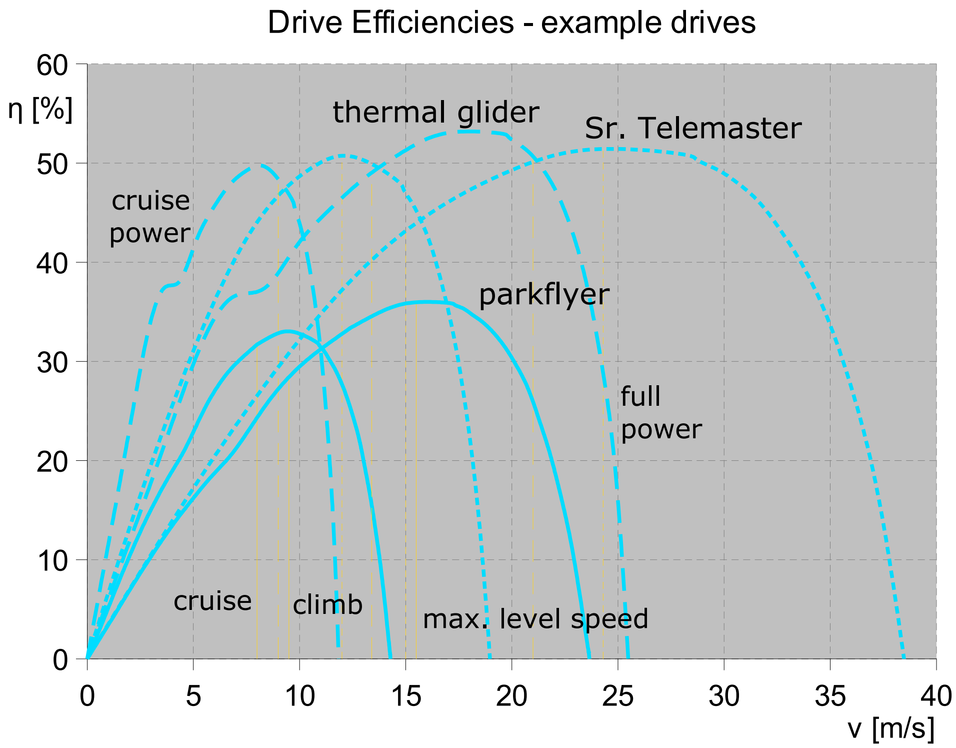
The rotational speed n curves begin with a more or less straight and constant part where blade stall occurs. It is followed by a parabolic increase over flight speed v. The lower specific rotational speed k_V and the bigger and "better" the drive, the smaller is the following increase ("unloading") – another characteristic of electric drives. Yet climb speed is always so low that little or no "unload" occurs. Maximum level speed or even "pitch speed", making for noticeable "unload", are not actually flown in practice.

Corresponding to the rotational speed n curves, the current I (amperage) curves begin with a more or less straight and constant part, here followed by an inverted parabola. Hence it is safe to assume that "static" amperage is never exceeded in flight, at least not significantly. Basically, it can be used as a criterion to choose a suitable ESC but only if maximum battery voltage is considered (for instance 4.2 V end-of-charge cell voltage for LiPo) instead of nominal voltage (3.7 V) like here. And the ESC must stand this amperage for some time because only slightly less is drawn during climb:



Electrical power P_{el} is amperage times battery voltage, which is constant. The parkflyer's as well as the thermal glider's drive have 8.4 V batteries. Their power curves happen to be drawn below their respective amperage curves due to the scaling of the two axes. Then again, the Sr. Telemaster's drive has a 14.8 V battery so its power curve is drawn well above its amperage curve.

As an exception, drive efficiencies η are shown for full-power as well as cruise-power settings (specified in the table above). Efficiencies are lower at cruise-power due to lower motor efficiency η_m at lower voltage, but the "better" the motor is the smaller is the difference:

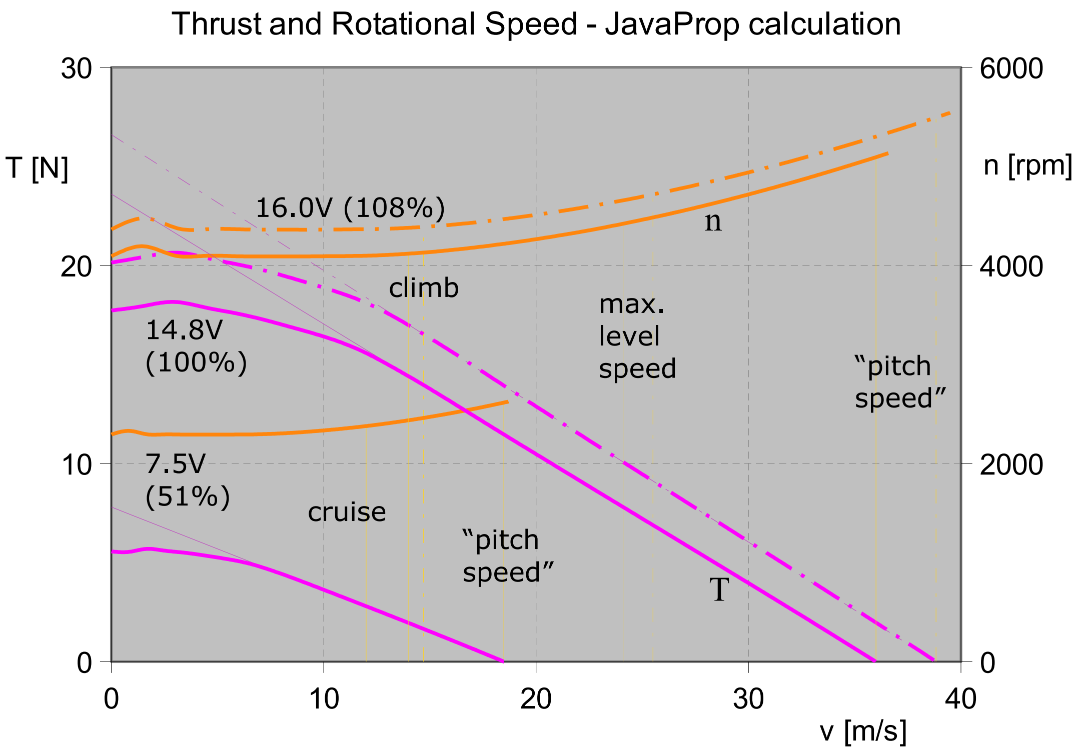


The parkflyer's drive is inefficient overall and its cruise-power efficiency would be even lower if it would not need a high power-setting (60%). By way of contrast, the thermal glider's drive is "better" but needs a low cruise-power setting (46%), which would be even lower if it would not be needed for flight speed with the moderate-pitch propeller. The Sr. Telemaster's drive is even "better" but its propeller's power loading is higher so its full-power peak efficiency is even lower than the thermal glider's.

As mentioned above, the parkflyer's and the Sr. Telemaster's drives are optimal for cruise flight. That means their drive efficiencies are close to or at peak, respectively, in cruise. In climb, though, they are far from peak and substantially lower than in cruise. Only at maximum level speed are their drive efficiencies at peak value, what is useless. By way of contrast, the thermal glider's drive is optimal for climb. There, its efficiency is closer to peak value and even nearly as high as the peak efficiency of the Sr. Telemaster's drive. Both its cruise speed and maximum level speed are higher than the respective speed of peak efficiency, though, so both efficiencies are lower.

Propeller Coefficient Comparison

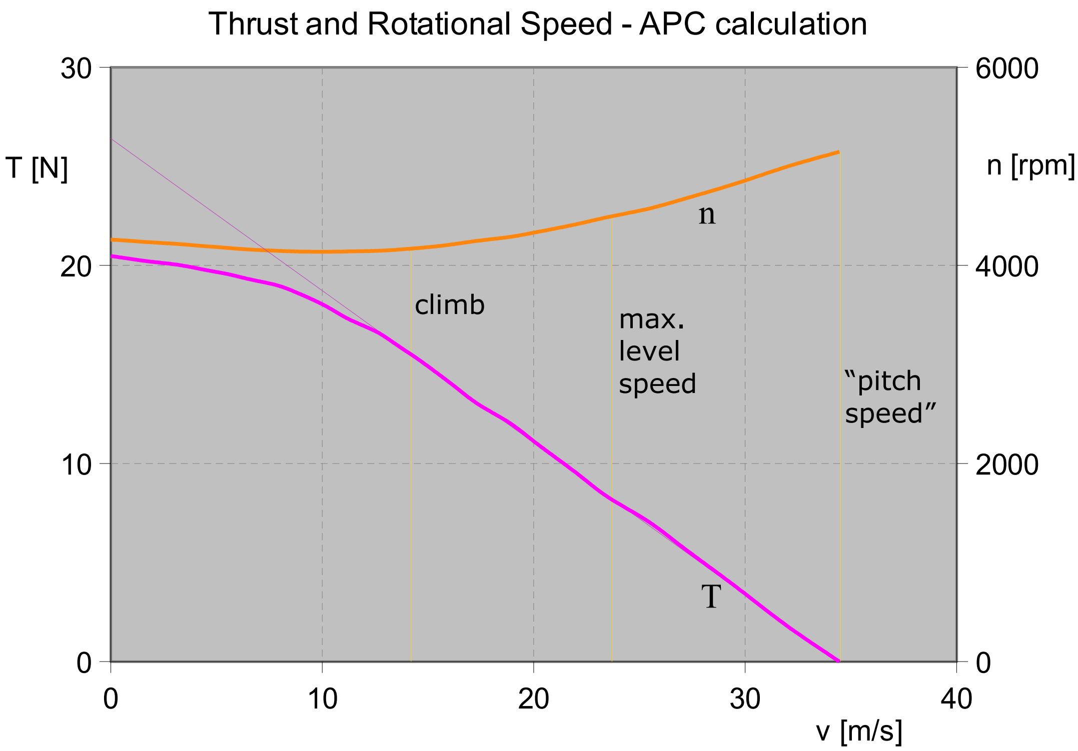
In the Propeller Illustration chapter, Coefficient Comparison section, different sets of coefficients for the Sr. Telemaster's APC 17x12 E propeller are compared. Here, each set is used for a drive calculation and the results are again compared. The consideration starts with the set of coefficients that had been calculated with JavaProp especially for the comparison. This case is actually known from the comparison in the previous chapter, so two variations of battery Voltage U_b have been added:



The first voltage variation is cruise-power setting, in this case 51% meaning 7.5 V equivalent battery voltage. The scaling of both the rotational speed n and the thrust T curves, that is their ends, is known from the main (first) example, but this (third) example explicitly shows that the whole thrust curve, including its straight part, is scaled so that the drive is slightly more elastic in cruise.

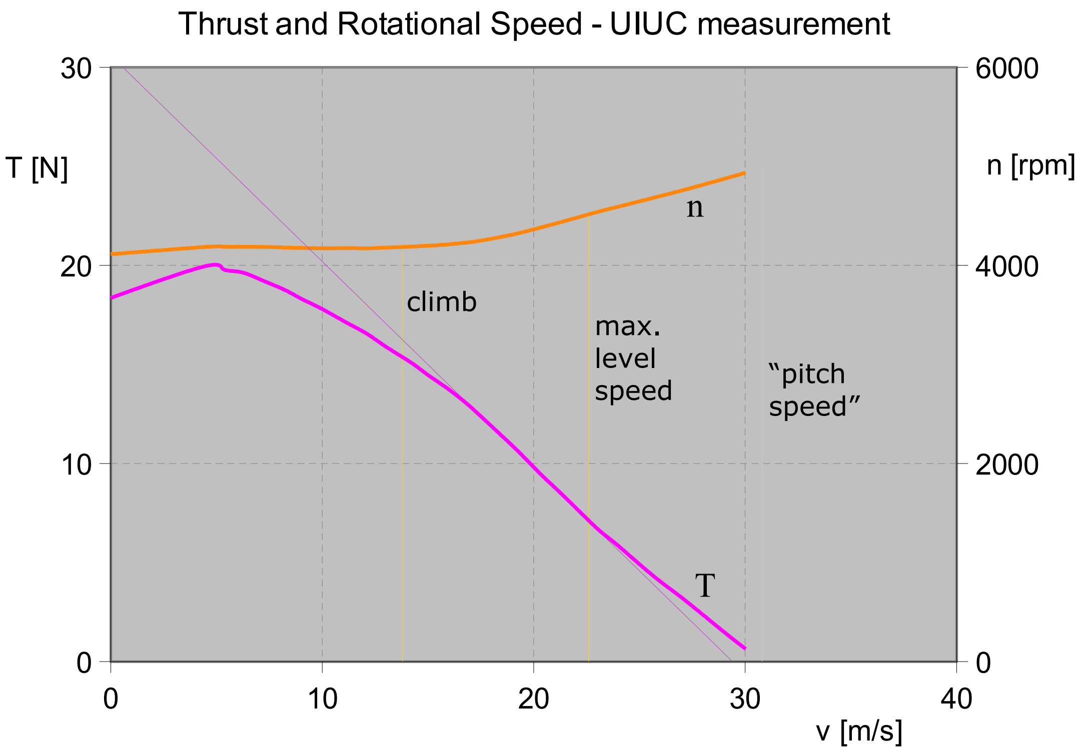
The second variation is assuming the battery is still at the beginning of discharge so cell voltage is 4.0 V and the 4S battery voltage is 16.0 V, equivalent to a 108% power setting (3.7 V nominal LiPo cell voltage defined to be 100%). This state of charge is reached after take-off with a fully charged battery and about half a minute of climb. Best climb speed v is slightly increased then, as is rotational speed n, but thrust T is over-proportionally increased. That means climb is noticeably more vigorous with a full battery.

Often, there are some warps in coefficient curves calculated with JavaProp, both the thrust coefficient c_T and the power coefficient c_P curves. By way of the drive calculations, these warps translate into mirror-inverted warps in the rotational speed n curve and corresponding warps in the thrust T curve. These warps occur only in the low advance ratio J and flight speed v ranges, respectively, where serious blade stall is predicted and the calculations are not reliable. *Static* thrust and rotational speed seem to be reliable in any case, though.



The calculation tool used by APC to compute propeller coefficients is considered as better than JavaProp. Still it yields similar thrust coefficient c_T curves with a straight part at higher advance ratios J where no blade stall is predicted. By way of the drive calculations, this as well translates into a straight part of the thrust T curve at corresponding higher flight speeds v. There are no warps in the low v range, though, just some waviness because the published coefficient values have only three digits.

The power coefficient c_P curves have a peculiarly increasing first part. It translates into a slightly decreasing first part of the rotational speed n curve, followed by the usual mild parabolic increase to maximum. Again there are no warps, just waviness.



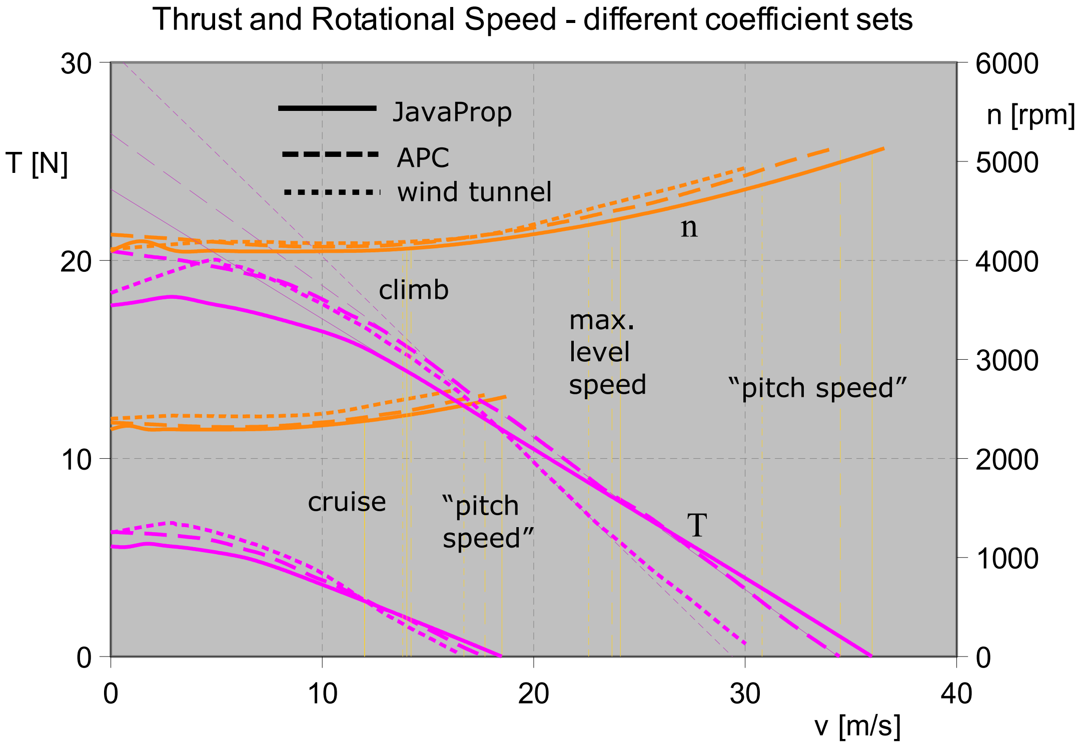
The propeller coefficients measured in a wind tunnel (at UIUC) result in coefficient curves with four distinct parts, each with its own slope and curvature. This seems to be a peculiarity of the APC Electric propellers since – for instance – coefficient curves for APC Sport propellers are less complex (see page 45 above).

Remarkably, the thrust T curve increases substantially from static thrust, perhaps because serious blade stall vanishes and the propeller has more and more "grip" in the air. Correspondingly, the rotational speed n curve slightly increases linearly.

Both curves' second part is as usual, meaning like seen before (JavaProp, APC calculations). The thrust curve resembles an inverted parabola and the rotational-speed curve is a horizontal straight line.

The curves' third part is as usual as well, now meaning the thrust curve is straight and the rotational-speed curve resembles a parabola. This is the part where the propeller coefficients are sensitive to rotational speed or Reynolds number, respectively. Propeller efficiency is close to maximum in this range, and horizontal flight speed is in this range, regardless of power setting. Hence this part of the thrust curve has been chosen as the straight part (see overlaid thinner and darker line) and because it is the least elastic one.

The thrust curve's fourth part seems to be straight as well but has less slope, that is it is more elastic. That is unusual, as well as the rotational-speed curve which seems to be straight here.



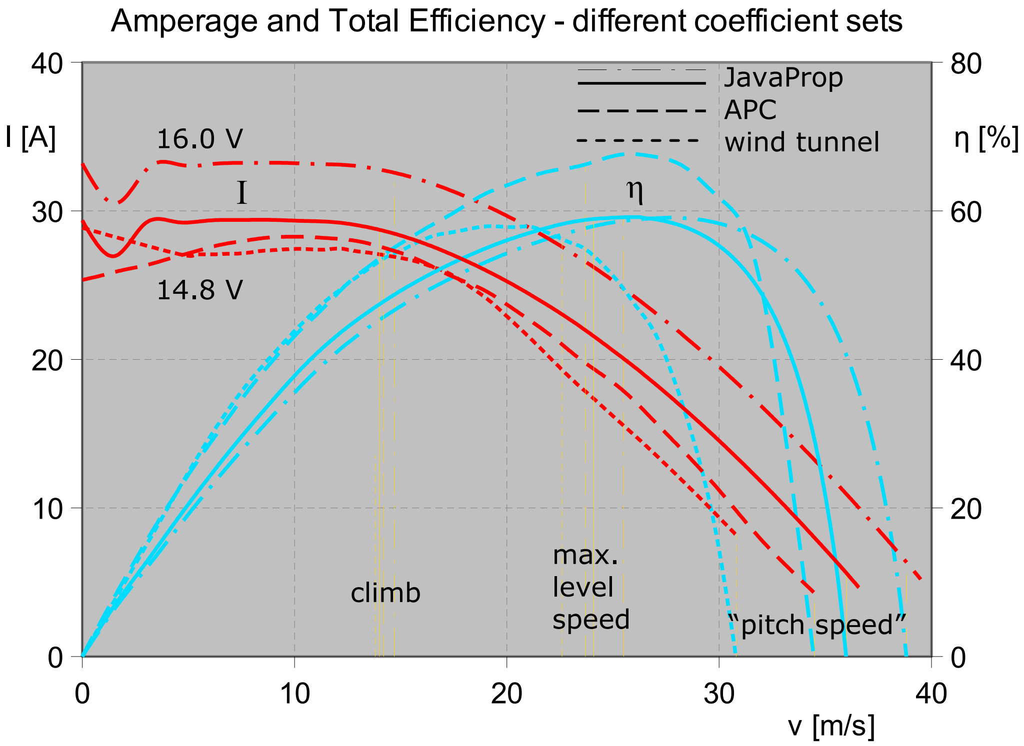
Finally, the three calculations are combined in one diagram and the cruise-power case is added to the full-power case. This puts the differences into perspective.

The full-power thrust T curves differ most. They intersect at speeds higher than climb speed, where JavaProp predicts less thrust than the other two calculations. The three curves' slope (elasticity) is similar at this speed. The measured coefficients make for the steepest slope, JavaProp for the most elastic in their respective characteristic flight speed range (see the overlaid thinner and darker lines). Anyway, climb speed is nearly equal in the three calculations and the thrust differences are comparatively small there and at slightly higher speeds. Even higher speeds (maximum level speed and "pitch speed") where the differences are bigger are not relevant in practice.

The cruise-power thrust T curves differ as well but they intentionally intersect at cruise speed, which was chosen to be 12 m/s in all three calculations. Hence the airframe's drag as well as the drive's thrust are equal, respectively.

The rotational speed n curves differ only slightly, curiously the cruise-power curves more than the full-power curves.

Compared to the difference between 3.7 V and 4.0 V cell voltage (the operational voltage range) shown three pages above, even the full-power thrust differences are smaller. At climb speed, JavaProp underestimates thrust compared to the better tool used by APC or even wind-tunnel measurements, all based on the lowest operational battery voltage. So we have a conservative estimate: If the calculated thrust is sufficient for the airplane then real thrust will be even more so.



The three calculations are again combined in one diagram to compare amperages and efficiencies, this time for full power only. A variation of the JavaProp case is added: the battery at 4.0 V cell voltage instead of 3.7 V (like four pages above).

While the wind tunnel measurements and the APC calculations are close to each other, JavaProp makes for the highest amperage. Again the differences between the three cases are smaller than the difference between charged and discharged battery. Since JavaProp overestimates amperage we have again a conservative estimate, usable to choose a suitable ESC if 4.0 V or even 4.2 V cell voltage is assumed in the calculation.

Best climb speed is nearly the same in all three cases but slightly higher with a charged battery. Total drive efficiency in climb is virtually the same in the wind tunnel and APC cases but substantially lower in the JavaProp case (even a bit lower with a charged battery). That is expectable – since thrust is lowest and amperage highest – and just another conservative estimate.

The APC calculations overestimate efficiency and corresponding flight speed at the same time while JavaProp overestimates flight speed only (each case compared to the wind tunnel case as reference). Then again, the difference between the curves for the two battery voltages is a characteristic of electric drives and correctly estimated.

This comparison suggests that the drive calculations are "on the safe side", regardless how "good" the propeller coefficients used for them really are.

Still the results don't necessarily match the real values. These drive calculations are not calibrated, or "tweaked" to some values measured on the real drive, from which they may differ more than from each other. Yet they are usable even without calibration, what has been validated by telemetry measurements.

To be continued...

Workflow

Spreadsheet Sequence

There are three modules of calculation, each in one of three interlinked spreadsheets:

1. Electrical characteristics of motor, speed controller, battery, cables, and plugs; as well as mechanical characteristics of motor and gear.
2. Aerodynamic and mechanical characteristics of the propeller.
3. Characteristics of the whole drive dependent on flight speed.

And there are another three, optional interlinked modules/spreadsheets:

4. Aerodynamic characteristics (coefficients) of the wing airfoil.
5. Aerodynamic characteristics of the whole airframe.
6. Performance characteristics of the airplane dependent on flight speed.

The former suffice to find a suitable drive or to compare different drives for a model. The drive characteristics are calculated for two cases: full power and cruise power. The latter not only show the performance characteristics of the whole airplane but thereby also help finding a suitable cruise power for the former.

It's typical to begin a complete calculation with nominal values specified by manufacturers or even with estimates and then gradually refine it by tweaking parameters until it is consistent. Basically, it can be calibrated with measured values. These may be published by component manufacturers, so the calculation is calibrated to the specific *type* of component. If the drive is already at hand, it can be measured for a calibration to the specific component *samples*, at first static for a safe maiden flight and finally in-flight just to be sure.

The spreadsheets are in one single file in the Microsoft® Office® Excel® XLS format, which should work in LibreOffice Calc as well (which has its own ODS format).

Example

A prototypical example is the drive of the author's Senior Telemaster *Plus* model airplane (the third drive in the comparisons). The calculation spreadsheets are available for download from the author's Web site. In this archive are several, slightly different calculations, especially one with propeller coefficients calculated by the manufacturer APC and one with coefficients measured by Brandt and Selig at UIUC. See the author's downloads page for more drive calculations.

There is also a comprehensive explanation of the calculation results – both flight performance and drive characteristics – on the author's review Web page for the Senior Telemaster *Plus* model airplane.

To be continued...