Analyzing Electric Model-Airplane Drives

Calculation of Drive Characteristics with Spreadsheet Tools

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About

At first glance, this document may look like a scientific one, but it is not meant to be one. It is meant for the technically inclined model-airplane flier who wants to know more about the characteristics of different electric drives (and different models, for that matter) – more than he can else know without having and just trying them all in the first place. Eventually, it just describes how the calculation spreadsheets work.

In the first instance though, it defines all equations needed to represent an electric drive and derives the basic solution as prerequisites. This is meant for those seriously inclined to understand how the calculation works. Of course, *some* technical and mathematical understanding or even expertise will help but should not be required. At least there are only simple differential equations and no integral equations, just plain algebra. It should be even possible to skip the explanations and derivations and just go to the description of workflow and calculation tools. At least the illustrations might be interesting, though.

Many definitions, lengthy explanations and derivations may contribute to a “scientific” look. But that and the phrasing are not meant to teach the reader but just to inform him how the spreadsheet calculations have been contrived. Any personal pronoun is avoided and “we” is used only in the sense of Pluralis Modestiae or Pluralis Auctoris (plural of modesty or author’s plural, do not seem to be named so in English), but in no case as Pluralis Majestatis (royal “we”). There should be no vowel omissions, no abbreviations, and no jargon either. However, that is all part of a quest for completeness, correctness, and conciseness.

The fonts are chosen for good on-screen readability. The pages may be displayed to fit the screen, in original size, or even enlarged – they should be easily readable in any case.

Reading this document on a display screen may be convenient because it can be searched for text strings and because there are some links to external documents in the World Wide Web.

Nevertheless, this document is well suited to printing on common DIN A4 paper with a monochrome printer. For the illustrating diagrams, a color printer would be better suited, though.

Introduction

This paper explains how common spreadsheet tools like Microsoft® Excel® or the free alternative OpenOffice™/LibreOffice Calc may be used to estimate the characteristics of an electric model-airplane drive. (It has been created and converted to PDF format with LibreOffice Writer, Math, and Draw.)

Using the word "estimate" is well-considered. No attempt is made to calculate the drive characteristics exactly or comprehensively. Quite the contrary, every possible simplification is used to define models and equations. There's nothing unusual about that since these simplifications are commonly used and their suitability is proven.

The achievable accuracy turns out to be well sufficient for the intended purpose. It's not about designing a drive optimized for a certain model but about *composing* a *suitable* drive from readily available components. These components – propeller, gear, motor, speed controller, and battery – are offered in various versions, sizes, and configurations. The point is that there are scales with certain value steps for the main features of drive components so there are only a few reasonable configurations in each case.

Propellers are offered in different kinds (sport, electric, parkflyer) as well as certain combinations of diameter and pitch. Electric motors come in certain combinations of power and specific speed (k_V). Both speed controllers and batteries just have to match the voltage (cell type and count), amperage, and capacity requirements of the chosen drive. In case a reduction gear is wanted there are usually very few choices.

Model-airplane manufacturers recommend a few reasonable motor/prop/battery combinations. And electric motor manufacturers recommend a few applications for each motor, meaning a class and weight of model as well as prop and battery. That all means that usually there are quite few choices of complete drives to be considered.

So using the word "analyzing" (electric drives) is well-considered as well. There is no way to put some desired parameters in and "calculate" the best-suited drive for a model. The only way to find it is c*omparing* a few promising configurations, maybe recommended by the model manufacturer or by the motor manufacturer, or even self-chosen. That is not too bad, though, since one may even want to compare a few variant drives which would give the model different characters.

In fact, this might be the main motivation to take the trouble of several tedious drive analyses. The manufacturer's recommendations will usually give a typical or "mainstream" drive and model character, but for instance less power but longer flight time might be wanted, or a drive optimized for cruise flight instead of climb. In such cases, one has to select and combine different drive components and check their suitability. Now the tedious procedure with several steps is worthwhile because different motors and propellers are mere interchangeable modules of the whole calculation.

Finally, the calculation may be calibrated once actual values of a real drive at hand can be measured. There are several parameters that are not exactly as specified or calculated. For instance, the sample strew of the motor magnets' field strength is said to be about 10%, making for correspondingly differing k_V values. By quite simple measurements the whole drive calculation can be calibrated ("tweaked") to a pretty good degree of accuracy (only a few percent error).

Since this is possible only after buying the drive, it is usually done just in edge cases, for instance before the maiden flight of a marginally powered airplane. Another important use case (which was the actual reason to develop the calculations described here) would be "building" a true-to-original simulator model.

Definitions

Units

Any specification of units is enclosed by brackets [].

Dimensionless variables are labeled with a null unit [-].

For convenience (no conversions), only coherent metric (SI) units are used.

Exception is rotational speed, which is specified as rpm [min^{-1}] – as usual – instead of angular speed ω in radians per second [rad/s]. Hence the conversion multiplier $2\pi/60$ is needed in some equations (2π radians per rotation, 60 seconds per minute).

No exception is made for efficiencies, which are often specified in [%], but are treated here as dimensionless ratios between 0 and 1 with a null unit [-].

Four natural unit conversions are used here, two mechanical and two electrical:

[N]=[kg m/s^2] and [W]=[Nm/s] [A]=[V/Ω] and [W]=[V A]

These conversions may be substituted and rearranged like ordinary equations.

Variables

R_b	[Ω]	resistance (impedance) of the battery
R_e	[Ω]	resistance (impedance) of controller (ESC), cables, and connectors
R_m	[Ω]	resistance (impedance) of the motor
R	[Ω]	resistance (impedance) of the whole system (total)
U_b	[V]	internal (no current) voltage of the battery
U_e	[V]	mean terminal voltage of the ESC as delivered to the motor
U_{mi}	[V]	mutual induction voltage of the rotating motor
I	[A]	actual current
I_{st}	[A]	stall current (rotor locked)
I_{0m}	[A]	idle current (no load) due to motor friction
I_{0g}	[A]	idle current (no load) due to gear friction
M_p	[Nm]	actual propeller moment (torque)
M_g	[Nm]	actual gear (propeller) shaft moment (torque)
M_m	[Nm]	actual motor shaft moment (external torque)
M_{st}	[Nm]	motor stall moment (total torque with rotor locked)
M_{0m}	[Nm]	motor idle moment (internal friction torque, constant)
M_{0g}	[Nm]	gear idle moment (internal friction torque, constant)
n	[s^{-1}]	actual drive speed (propeller revolutions per second)
n_p	[min^{-1}]	actual propeller speed (revolutions per minute)
n_g	[min^{-1}]	actual gear output (propeller) shaft speed
n_m	[min^{-1}]	actual gear input (motor) shaft speed
n_0	[min^{-1}]	drive idle (no-load) speed
n_{0g}	[min^{-1}]	gear shaft (drive) idle (no-load) speed
n_{0m}	[min^{-1}]	motor shaft idle (no-load) speed

P_{thrust}	[W]	actual propeller (output) thrust power
P_{shaft}	[W]	actual propeller (input) shaft power
P_{mech}	[W]	actual gear output shaft (mechanical) power
P_m	[W]	actual motor output shaft (mechanical) power
P_{el}	[W]	actual system input (battery) electric power
η	[-]	total system efficiency ("eta", drive and propeller)
η_m	[-]	motor efficiency (including battery and ESC)
η_g	[-]	gear efficiency
η_p	[-]	propeller efficiency
η_d	[-]	drive efficiency (motor and gear)
k_V	[min^{-1}/V]	specific rotational speed (rpm per Volt)
k_A	[A/min^{-1}]	specific current (Ampere per rpm, *negative* value)
k_M	[Nm/A]	specific moment (torque per Ampere)
i_g	[-]	gear reduction ratio (e.g. 4.4 for a 4.4:1 gear)
D	[m]	propeller diameter
v	[m/s]	flight speed
J	[-]	advance ratio
c_T	[-]	propeller thrust coefficient
c_M	[-]	propeller moment (torque) coefficient
c_P	[-]	propeller power coefficient
g	[kg m/s^2]	gravity acceleration (standard is 9.81)
ρ	[kg/m^3]	density of air ("rho", standard is 1.226)
ν	[m^2/s]	kinematic viscosity of air ("nue", standard is 0.00001464)
a	[m/s]	speed of sound (standard is 340.29)
π	[-]	number "pi" (3.14159)

Transformations

Specific motor moment (torque) k_M is simply specific speed k_V transformed, as well as propeller moment coefficient c_M is simply power coefficient c_P transformed:

$$k_M = \frac{60}{2 \cdot \pi \cdot k_V} \qquad c_M = \frac{c_P}{2 \cdot \pi}$$

Inconsistencies

The *propeller's* rotational speed n has the unit [s^{-1}], not [min^{-1}]. That is due to the tool used for calculating the propeller coefficients, which requires this unit for the way the advance ratio is defined.

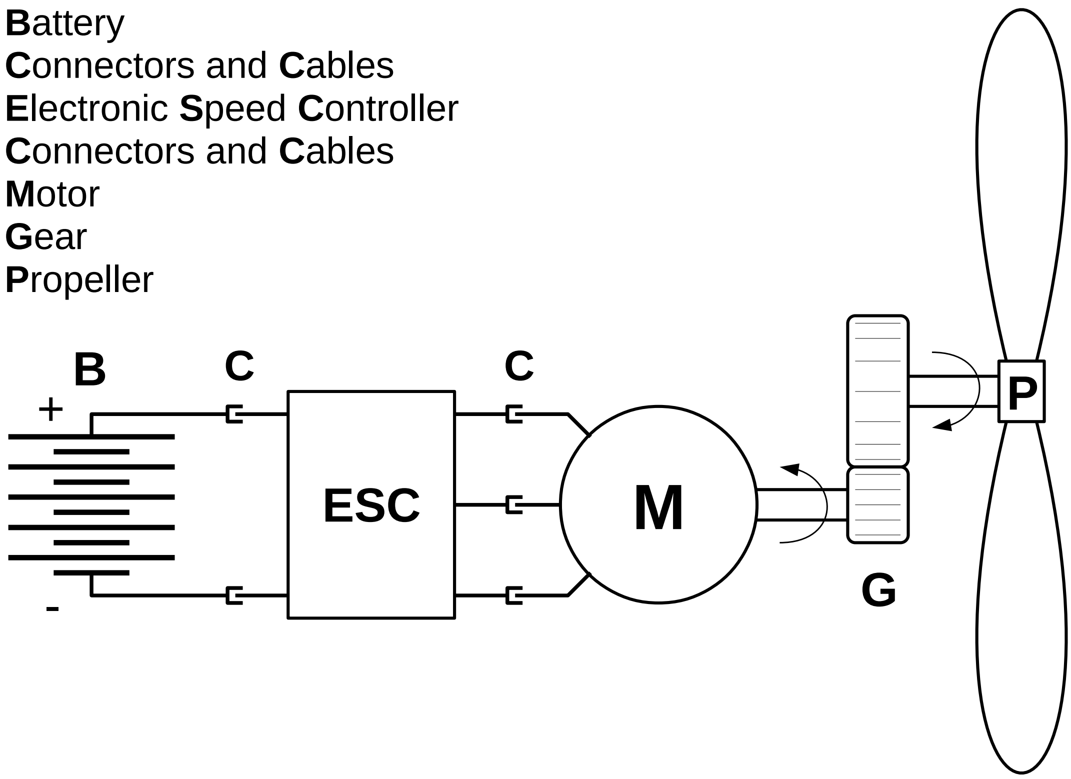
In the chapter Basic Solution, in the sections following Specific Speed/Moment, all rotational speeds are assumed to have this unit. That is not consistent with the definitions above. In the section Mechanical-Aerodynamic Conversion, n in [s^{-1}] is consistently used, though.

Simplified Drive Model

Modeling the drive means defining equations which describe it's behavior. As usual, one may discern two steps of modeling: abstraction and relaxation. In the first step – abstraction – all aspects relevant to the task are identified and all others are omitted. Usually there is still no way to draft equations, so in the second step – relaxation – even relevant aspects are omitted or at least rendered simpler than they really are. Relaxation is carried as far as necessary to find a solution in equation form.

Generic Drive Model

So we start by defining an abstract, generic model of a whole drive. The first relevant aspect is to compose the drive from common interchangeable components:



Each component is seen as a “black box” with interfaces to other components. The electrical and/or mechanical properties constitute a component’s behavior at these interfaces, which has to be described in equation form.

The *battery* may have various numbers of cells, various capacities as well as loads (C rate), and various cell types (voltages). This is simply described as particular values of corresponding variables, for instance a 5s1p 5000 mAh 30C LiPo battery.

The *ESC (electronic speed controller)* has to match the type of motor (brushed or brushless), it's size (power), and the battery’s voltage. It feeds the motor with varying electric power. For convenience, connectors and cables are assigned to the ESC.

The *motor* converts electrical to mechanical power. It may be brushed or brushless, inrunner or outrunner, and have various speeds and sizes/power.

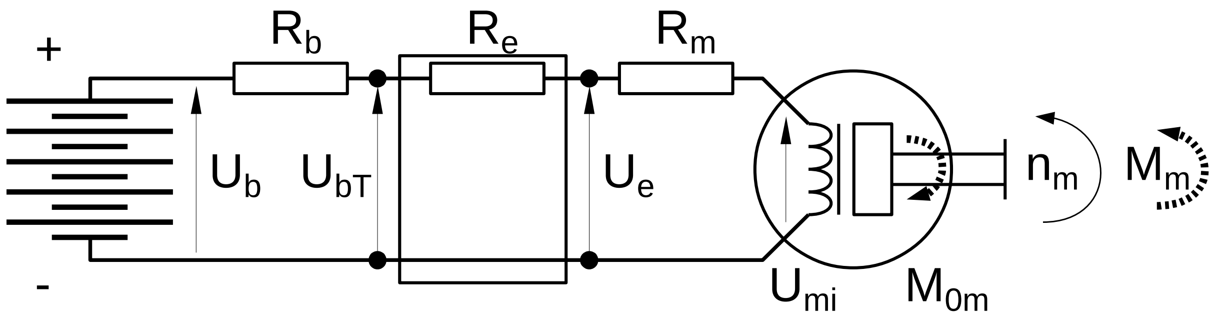
The *gear* transforms the mechanical power to different rpm and torque. It may be a spur/ring/planetary gear, and have various transmission ratios, sizes, and efficiencies.

Finally, the prop converts the mechanical power to aerodynamic thrust and torque. There are different shapes and number of blades, diameter and pitch, and folding.

Electrical-Mechanical Conversion

Electromechanical systems are modeled in the form of an equivalent circuit diagram. It specifies the system's characteristics and their interrelations.

Model-airplane motors are permanent-magnet DC motors, and a brushless motor is essentially the same, just with the mechanical commutator (collector/brushes) replaced by the ESC. (The links lead to Wikipedia.) That is why there are only one coil and two lines in the diagram's motor symbol, and why it is a DC circuit diagram. All effects of alternating and pulsing current are neglected or replaced by resistances, respectively. That is a very useful and still acceptable simplification.



The battery provides an "internal" voltage, which depends on type and number of cells. We use the standard or nominal voltage of the cell type at hand, that is 3.7 V for LiPo, 3.3 V for LiFePo, and 1.2 V for NiMH or NiCd. According to Ohm's law, the battery's terminal voltage is lower than the internal voltage while any current is flowing because there is some (complex) internal impedance in a battery, here depicted by a simple (constant) resistor:

$$U_{bT} = U_b - R_b \cdot I$$

The ESC reduces the voltage as well due to its internal resistance, which includes all connector and cable resistances in our simplified model. Beyond that, its "throttle" function is seen here simply as further reduction of (mean) voltage. At WOT (wide open throttle), the ESC delivers a slightly reduced voltage to the motor:

$$U_e = U_{bT} - R_e \cdot I$$

Of course, also the motor has an ohmic resistance, which is a replacement for the real ohmic resistance as well as for complex electric and magnetic impedances. In the simplified model, the motor coil sees a slightly reduced voltage:

$$U_{mC} = U_e - R_m \cdot I$$

Further simplifying our model, we assume that the ESC's "throttle" function reduces *the battery's* internal voltage. That allows to sum up one single ohmic resistance:

$$U_{mC} = U_b - (R_b + R_e + R_m) \cdot I = U_b - R \cdot I$$

This voltage applied to the motor coil is antagonized by an opposing voltage that is literally generated in the spinning motor by so-called mutual induction and sometimes also aptly called generator voltage. It is proportional to rotational speed and here is where the k_V value (*specific* rotational speed) comes into play:

$$U_{mi} = n_m / k_V \quad \text{and} \quad U_{mC} = U_{mi} \quad \text{hence} \quad k_V = n_m / U_{mi} \quad \text{or} \quad k_V = n_m / U_{mC}$$

This equation shows that k_V essentially tells how fast the motor spins proportional to the effective voltage applied. That is one of the main motor characteristics, depending on number of poles, number of windings, and the motor's geometry/size.

Disregarding any current, the voltage effective in the motor is the battery voltage reduced by the mutual induction voltage:

$$U_{eff} = U_b - U_{mi} = U_b - n_m / k_V$$

Then, according to Ohm's law, the current flowing through the motor coil and the whole system is the ratio of effective voltage and total resistance:

$$I = U_{eff} / R = (U_b - n_m / k_V) / R$$

If the motor is stalled (blocked), there is no rotation, hence no mutual induction, and current depends solely on ohmic resistance:

$$I_{st} = U_b / R$$

For convenience, we will use current (amperage) expressed as directly dependent on rotational speed. That is possible by means of *specific* current k_A, which (as a *negative* value) tells how much current flows *inversely* proportional to rotational speed:

$$I = I_{st} + k_A \cdot n_m \quad \text{what makes} \quad k_A = \frac{-1}{R \cdot k_V} \quad \text{(I and } I_{st} \text{ substituted)}$$

k_V can be transformed into specific moment (torque) k_M, which directly (hence conveniently) tells how much moment (torque) is *produced* proportional to current:

$$M_m = I \cdot k_M$$

However, this equation is provisional. In addition to the electric losses, there are complex mechanic and magnetic losses in the motor. For simplicity's sake, they are represented by a constant internal friction moment that *re*duces the torque output:

$$M_m = I \cdot k_M - M_{0m}$$

An idling motor doesn't produce any output torque, but it still has to overcome the internal friction moment what requires a corresponding idle current:

$$M_{0m} = I_{0m} \cdot k_M \quad \text{hence} \quad I_{0m} = M_{0m} / k_M \quad \text{and} \quad M_m = (I - I_{0m}) \cdot k_M$$

Even in case of stall (blocked rotor) this friction (idle) moment is assumed active. Yet for convenience we *define* the stall moment as total torque produced internally:

$$M_{st} = I_{st} \cdot k_M$$

By the way, sometimes the friction moment is omitted in drive calculations. That spoils the calculation, which is actually simple: voltage makes for speed (rpm), and amperage makes for moment (torque), both proportionally and interdependently.

However, now the motor's output (mechanical) power can be derived from torque and speed, and its input (electrical) power from current and (battery) voltage. Additionally, the simple terms are substituted with complex ones which contain only specified constants and the rotating speed as sole variable, just to demonstrate that:

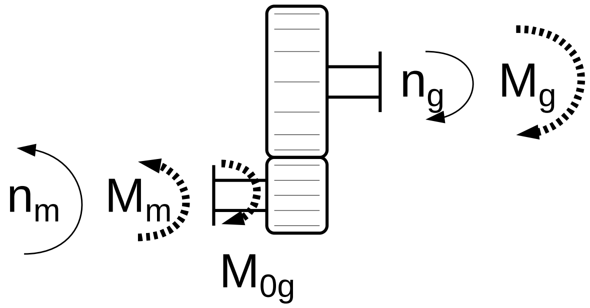
$$P_m = M_m \cdot \frac{2 \cdot \pi}{60} \cdot n_m = \frac{-1}{R \cdot k_V^2} \cdot n_m^2 + \frac{U_b - R \cdot I_{0m}}{R \cdot k_V} \cdot n_m \qquad P_{el} = I \cdot U_b = \frac{-U_b}{R \cdot k_V} \cdot n_m + \frac{U_b^2}{R}$$

Because the system's total resistance was used in the calculations so far, this resistance is included in the efficiency as well. So motor efficiency actually means drive efficiency, still not including the gear (whose efficiency is later included by multiplication). So the drive's efficiency is (provisionally) just the ratio of motor powers out/in:

$$\eta_m = \frac{P_m}{P_{el}} \quad \text{(We spare us substituting the simple terms.)}$$

Mechanical-Mechanical Conversion

Mechanical systems are modeled in the form of a schematic sketch or, more specifically, a free body diagram. It shows several connected bodies or a single body with all of their applied forces and moments, that is their "interface". The gear is one of the bodies and is a simple transmission or rpm/torque transformer, respectively, between motor and propeller.



The most obvious gear property is its transmission ratio. In the case of model-airplane drives, it is always a reduction ratio. That means a quite high motor speed is reduced to a lower propeller speed. Conversely, a quite low motor moment is transformed to a higher propeller torque. Both speed and torque *directions* may be reversed – by a spur gear like in the sketch – but that does not matter in our calculations. However, the second equation is provisional:

$$n_g = n_m / i_g \qquad M_g = M_m \cdot i_g$$

After all, the gear makes for some power losses. Obviously, rotational speeds are mechanically fixed so the losses appear as reduction of torque. That is quite plausible since the losses stem from friction. We can see this in two extremely simple ways: constant or proportionally dependent on rotational speed. Either way, the moments are reduced and we just assume (define) it is the input moment:

$$M_g = (M_m - M_{0g}) \cdot i_g$$

Difference is that *in this first case* the friction moment as an absolute value has to be measured or guessed, what may be hard. Easier may be just estimating and later "tweaking" a gear efficiency as a relative value (*second case*):

$$M_{0g} = (1 - \eta_g) \cdot M_m \quad \text{or directly} \quad M_g = M_m \cdot \eta_g \cdot i_g$$

Probably a combination of both ways (and even non-proportional speed-dependent friction) would be correct, but for simplicity's sake one of the two ways is chosen. *In any case*, gear friction can be treated like internal motor friction so a gear friction moment requires a corresponding current just like the motor friction moment.

That solves the problem of calculating a gear efficiency *in the first case* by calculating a total mechanical drive power and efficiency:

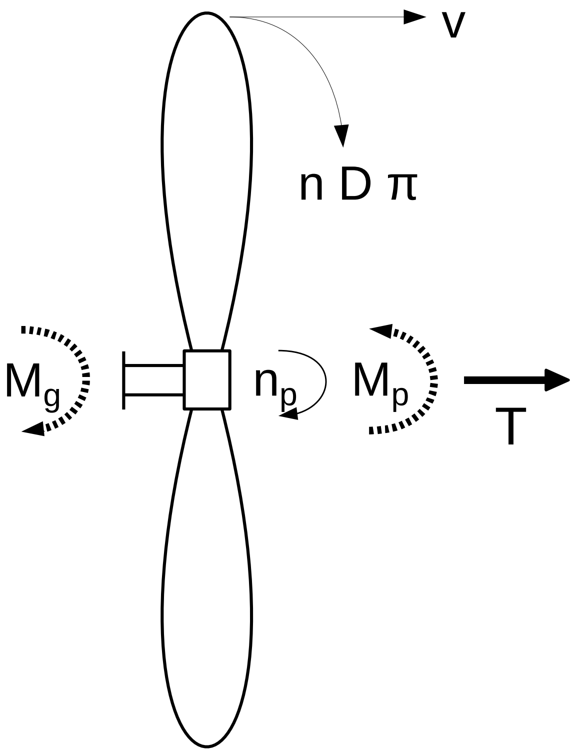
$$I_{0g} = M_{0g} / k_M \quad \text{makes} \quad P_{mech} = M_g \cdot \frac{2 \cdot \pi}{60} \cdot n_g = -\frac{i_g^2}{R \cdot k_V^2} \cdot n_g^2 + \frac{U_b - R \cdot (I_{0m} + I_{0g})}{R \cdot k_V} \cdot i_g \cdot n_g$$

Again *in any case*, the drive's efficiency is finally the ratio of mechanical power output to the gear shaft and electrical power input from the battery:

$$\eta_d = \frac{P_{mech}}{P_{el}} \quad \text{or else in our \textit{second case}:} \quad \eta_d = \eta_m \cdot \eta_g$$

Mechanical-Aerodynamic Conversion

The next and last body, the propeller, is a quite complicated component aerodynamically, so we will have to rely on a correspondingly complex, specialized tool to calculate the moment (torque) and other coefficients.



Propellers are usually characterized by dimensionless coefficients, which are valid for a certain geometric shape, regardless of size. They are in some way related to the propeller's diameter D as a common measure of size, as well as to rotational speed n [s^{-1}]. There are three characteristics (T, M, P) and corresponding coefficients (c_T, c_M, c_P):

Thrust $T = c_T \cdot \rho \cdot n^2 \cdot D^4$

Moment (torque) $M_p = c_M \cdot \rho \cdot n^2 \cdot D^5$

Power $P_{shaft} = c_P \cdot \rho \cdot n^3 \cdot D^5$

Now shaft power is also:

$$P_{shaft} = 2 \cdot \pi \cdot n \cdot M_p = 2 \cdot \pi \cdot c_M \cdot \rho \cdot n^3 \cdot D^5$$

what makes $c_M = \frac{c_P}{2 \cdot \pi}$

In some way, these characteristics also depend on flight speed. So the coefficients, as dimensionless values, must be related to a kind of dimensionless flight speed as well as rotational speed. That is the advance ratio, which is meant to be the ratio of flight speed v [m/s] and circumferential blade tip speed:

$\lambda = \frac{v}{n \cdot D \cdot \pi}$ but actually used is this slightly simpler definition: $J = \frac{v}{n \cdot D}$

Anyway, the tool mentioned above delivers the coefficients over the whole range of advance ratios, or from zero speed to top speed, as it were. So it is possible to calculate the mechanical power needed to drive the propeller (equation above) and the thrust power produced by it:

$P_{thrust} = T \cdot v = J \cdot c_T \cdot \rho \cdot n^3 \cdot D^5$ because $v = J \cdot n \cdot D$

The propeller's efficiency is the ratio of these two powers:

$\eta_p = \frac{P_{thrust}}{P_{shaft}}$ what makes it directly (dimensionless): $\eta_p = J \cdot \frac{c_T}{c_P}$

And the total system efficiency is the ratio of thrust power and electrical power:

$\eta = \frac{P_{thrust}}{P_{el}}$ or else: $\eta = \eta_m \cdot \eta_g \cdot \eta_p$

See Martin Hepperle's JavaProp Users Guide.

Basic Solution

Approach

All the equations presented above show that a drive's behavior can be described as dependent on several given constants and one single variable – *rotational* speed. That includes even the propeller, so eventually the drive's behavior can be described over a whole *flight* speed range from "static" (zero speed) to "pitch speed" (zero thrust).

Of course, this was intended, but it is only the basis for a solution in equation form. There has to be – and now can be – one single equation that delivers the rotational speed of the whole drive including propeller. To develop this equation, we have to equate something of the drive with the same thing of the propeller.

We want to equate the motor/gear (drive) torque with the propeller torque (moment), both dependent on rotational speed (rpm). We prefer the *torques* to get only a second-order polynomial. Equating motor/gear and propeller *power* would give a third-order polynomial, which would be unnecessarily complicated.

Accordingly, the term "constant" means independent of rpm; "decrease" or "increase" mean change with rpm or even rpm squared, respectively.

In the following derivation, the given k_V value is not used but the k_M and k_A values instead because that is more convenient. As mentioned above and proven below, k_M is k_V transformed, and k_A is easily calculated from two given constants.

There are those two extremely simple ways to see gear losses: constant, or proportional to rpm. The former makes for more simple and obvious combined constants while the latter seems to be more practical. Both ways are presented here for comparison, but we will stick to the more practical way after that.

We get only a *basic* solution in the end insofar as it just gives rotational speed dependent on propeller power coefficient. However, the specialized propeller analysis tool used here delivers this and other coefficients for the whole range of possible advance ratios. That is equivalent to flight speed range, so deriving all other characteristics for this range is easy then.

Specific Speed/Moment

The transformation of specific speed k_V into specific moment (torque) k_M has not been derived yet. It may be appropriate to make up for that before using it in the solution.

We consider only the conversion of electrical into mechanical power "inside" the motor but omit (disregard) electrical losses "before" and mechanical losses "after" it. The constants define "internal" voltage and moment, respectively. Multiplying them by current and speed, respectively, yields "internal" powers. Equating mechanical with electrical power clearly shows the transformation in question (Wikipedia).

$$U_{mi} = \frac{n_m}{k_V} \qquad P_{el} = U_{mi} \cdot I = \frac{n_m}{k_V} \cdot I$$

$$M_m = I \cdot k_M \qquad P_{mech} = M_m \cdot \frac{2 \cdot \pi}{60} \cdot n_m = I \cdot k_M \cdot \frac{2 \cdot \pi}{60} \cdot n_m$$

$$P_{mech} = P_{el} \qquad I \cdot k_M \cdot \frac{2 \cdot \pi}{60} \cdot n_m = \frac{n_m}{k_V} \cdot I \qquad k_M = \frac{60}{2 \cdot \pi \cdot k_V}$$

Drive Torque 1

Quite simple and obvious combined constants K_1 and K_2 result for the drive (motor and gear) from assuming constant gear friction (*first case* above).

Torque comes from current in the electric motor, so this to begin with:

$$I = I_{st} + k_A \cdot n_m = I_{st} + k_A \cdot i_g \cdot n_g \qquad \text{because } n_m = i_g \cdot n_g$$

That makes the drive's torque (moment) dependent on rotational speed (rpm):

$$M_g = k_M \cdot I - M_{0m} - M_{0g} = (I - I_{0m} - I_{0g}) \cdot k_M \qquad \text{because } M_{0m/g} = k_M \cdot I_{0m/g}$$

$$M_g = (I_{st} - I_{0m} - I_{0g} + k_A \cdot i_g \cdot n_g) \cdot k_M \qquad \text{I substituted with equation above}$$

$$M_g = I_{st} \cdot k_M - I_{0m} \cdot k_M - I_{0g} \cdot k_M + k_A \cdot k_M \cdot i_g \cdot n_g \qquad \text{expanded}$$

Combining the constants makes things clearly arranged:

$$K_1 = (I_{st} - I_{0m} - I_{0g}) \cdot k_M \qquad \text{torque output when stalled [Nm]}$$

$$K_2 = 60 \cdot k_A \cdot k_M \cdot i_g \qquad \text{torque decrease with speed [Nm/s}^{-1}\text{]}$$

$$M_g = K_2 \cdot n_g + K_1 \qquad \text{speed } n_g \text{ in rotations per } \textit{second} \text{ [s}^{-1}\text{]}$$

Drive Torque 2

Assuming gear friction in the form of gear efficiency, that is proportional to moment and hence inversely proportional to rotational speed (*second case* above), results in slightly more complicated combined drive constants K_1 and K_2. Yet it is a more practicable way than the first one.

Torque comes from current in the electric motor, so this to begin with:

$$I = I_{st} + k_A \cdot n_m = I_{st} + k_A \cdot i_g \cdot n_g \qquad \text{because } n_m = i_g \cdot n_g$$

That makes the motor torque (moment) dependent on rotational speed (rpm):

$$M_m = k_M \cdot I - M_{0m} = (I - I_{0m}) \cdot k_M \qquad \text{because } M_{0m} = k_M \cdot I_{0m}$$

$$M_m = (I_{st} - I_{0m} + k_A \cdot i_g \cdot n_g) \cdot k_M \qquad \text{I substituted with equation above}$$

Introduce gear efficiency and expand the equation:

$$M_g = M_m \cdot \eta_g \cdot i_g$$

$$M_g = (I_{st} - I_{0m} + k_A \cdot i_g \cdot n_g) \cdot k_M \cdot \eta_g \cdot i_g \qquad M_m \text{ substituted with equation above}$$

$$M_g = I_{st} \cdot k_M \cdot \eta_g \cdot i_g - I_{0m} \cdot k_M \cdot \eta_g \cdot i_g + k_A \cdot k_M \cdot \eta_g \cdot i_g^2 \cdot n_g \qquad \text{expanded}$$

Combining the constants makes things clearly arranged:

$$K_1 = (I_{st} - I_{0m}) \cdot k_M \cdot \eta_g \cdot i_g \qquad \text{torque output when stalled [Nm]}$$

$$K_2 = 60 \cdot k_A \cdot k_M \cdot \eta_g \cdot i_g^2 \qquad \text{torque decrease with speed [Nm/s}^{-1}\text{]}$$

$$M_g = K_2 \cdot n_g + K_1 \qquad \text{speed } n_g \text{ in rotations per } \textit{second} \text{ [s}^{-1}\text{]}$$

Propeller Torque

Propeller moment (torque) depends on rotational speed (rps) in any case:

$$M_p = c_M \cdot \rho \cdot D^5 \cdot n_p^2$$ speed n_p in rotations per *second* [s^{-1}]

Again combining the constants makes:

$$K_3 = \rho \cdot D^5$$ torque increase with speed [Nm/s^{-2}]

$$M_p = c_M \cdot K_3 \cdot n_p^2$$ c_M is not constant!

Formal Solution

Equating propeller torque with drive torque is our approach:

$$M_p = M_g = M$$ (and of course $n_p = n_g = n$)

This is really clearly arranged and easy:

$$c_M \cdot K_3 \cdot n_p^2 = K_2 \cdot n_g + K_1$$ substituted

$$c_M \cdot K_3 \cdot n^2 + (-K_2) \cdot n + (-K_1) = 0$$ rearranged (normalized)

There are standard solutions for such a second-order polynomial:

$$\Delta = 4 c_M K_3 (-K_1) - (-K_2)^2$$ discriminant

The discriminant is negative for all values of c_M, so there are two possible solutions:

$$n_{1,2} = \frac{-(-K_2) \pm \sqrt{(-K_2)^2 - 4 c_M K_3 (-K_1)}}{2 c_M K_3}$$ possible solutions

n_1 (with positive square root) is the correct solution since n_2 would be negative.

That was the formal derivation of the basic solution, but we can write it simpler now:

$$n = \frac{K_2 + \sqrt{K_2^2 + 4 c_M K_3 K_1}}{2 c_M K_3}$$ solution

See Quadratic Formula at Wikipedia. Dimensional analysis is done in the next section.

Applicable Solution

Finally, we want to make the polynomial constants K_1, K_2, and K_3 directly and exclusively depend on given constants, which usually are U_b, R, I_{0m}, k_V, i_g, η_g, ρ, D, and c_P. That also subtly modifies the second-order polynomial they belong to:

$$c_P \cdot K_3 \cdot n^2 - K_2 \cdot n - K_1 = 0 \quad \text{[Nm]} \qquad (c_M \text{ substituted with } c_P)$$

K_1 is the polynomial's constant term, and since we equated drive and propeller moment, it must be a moment as well. In fact, it is the drive's torque output when stalled, is a fixed positive number, and has the unit [Nm]:

Substituting $I_{st} = \frac{U_b}{R}$ and $k_M = \frac{60}{2 \cdot \pi \cdot k_V}$ makes

$$K_1 = (I_{st} - I_{0m}) \cdot k_M \cdot \eta_g \cdot i_g = \left(\frac{U_b}{R} - I_{0m} \right) \cdot \frac{60}{2 \cdot \pi \cdot k_V} \cdot \eta_g \cdot i_g \quad \text{[Nm]}$$

K_2 is the term proportional to rotational speed n [s^-1], so it must be moment change. In fact, it is the drive's torque-output decrease and therefore a negative number. Because rotational speed is rotations *per second* here, the unit is [Nm/s^-1]:

Substituting $k_A = \frac{-1}{k_V \cdot R}$ and $k_M = \frac{60}{2 \cdot \pi \cdot k_V}$ in the K_2 equation and expanding *the formal solution* (above) with ½ makes

$$K_2 = 60 \cdot k_A \cdot k_M \cdot \eta_g \cdot i_g^2 = \frac{1}{2} \cdot \frac{60}{2 \cdot \pi} \cdot \frac{-60}{R \cdot k_V^2} \cdot \eta_g \cdot i_g^2 = \frac{-900}{\pi \cdot R \cdot k_V^2} \cdot \eta_g \cdot i_g^2 \quad \text{[Nm/s}^{-1}\text{]}$$

K_3 is the term proportional to rotational speed squared n^2 [s^-2], so it is a moment change as well. It is the propeller's torque-input increase and therefore a positive number. It comes from aerodynamic lift and drag, which in turn increase with airspeed squared, so the unit has to be [Nm/s^-2]:

Substituting $c_M = \frac{c_P}{2 \cdot \pi}$ *in the polynomial* (above) makes

$$K_3 = \rho \cdot D^5 = \frac{\rho \cdot D^5}{2 \cdot \pi} \quad \text{[Nm/s}^{-2}\text{]}$$

Now we can write the solution in a form practical for use:

$$n = \frac{K_2 + \sqrt{K_2^2 + c_P K_3 K_1}}{c_P K_3} \quad \text{[s}^{-1}\text{]} \qquad \text{(expanded with ½)}$$

Motor/Gear Illustration

Characteristic Speeds and Quantities

To illustrate the motor/gear combination's characteristics dependent on rotational speed, we need equations for some *characteristic* rotational speeds as well.

The rotational speed of a stalled motor is zero by definition:

$$n_{st} = 0$$

Maximum rotational speed is "theoretical" or "ideal" because it could be reached only if there were no friction. It is where current I is (or would be) zero. We use two equations from the Electrical-Mechanical Conversion section to substitute variables in a current equation from the Drive Torque 2 section. Equating this with zero gives maximum rotational speed $n_{g\,max}$:

$$I_{st} = \frac{U_b}{R} \qquad k_A = \frac{-1}{R \cdot k_V} \qquad I = I_{st} + k_A \cdot i_g \cdot n_g = \frac{U_b}{R} - \frac{i_g \cdot n_g}{R \cdot k_V} = 0 \qquad n_{g\,max} = U_b \cdot \frac{k_V}{i_g}$$

The point (rotational speed) where the moment (torque) output is zero, is called idle (or no-load) speed n_0. In the section Electrical-Mechanical Conversion, we had an equation for I dependent on rotational speed. When idle, current flows only to overcome internal motor friction, that is idle current I_{0m}. By definition, there is no gear friction when idle (no moment output) in our *second case* (gear efficiency η_g is specified). So just equating current I with motor idle current I_{0m} gives idle speed n_0:

$$I = \left(U_b - \frac{i_g \cdot n_g}{k_V}\right) \cdot \frac{1}{R} = I_{0m} \quad \text{rearranged results in} \quad n_0 = n_{0g} = \left(U_b - R \cdot I_{0m}\right) \cdot \frac{k_V}{i_g}$$

Next is the point of maximum mechanical power output P_{mech}. There is a maximum because P_{mech} is a negative (inverted) parabola as shown in Mechanical-Mechanical Conversion. Here we write the equation in the form for our *second case*. Then we differentiate P_{mech} with respect to n_g and equate the result with zero. That reveals the *position* of maximum mechanical power output being at half idle speed:

$$P_{mech} = M_g \cdot \frac{2 \cdot \pi}{60} \cdot n_g = M_m \cdot \eta_g \cdot i_g \cdot \frac{2 \cdot \pi}{60} \cdot n_g = -\frac{\eta_g \cdot i_g^2}{R \cdot k_V^2} \cdot n_g^2 + \left(U_b - R \cdot I_{0m}\right) \cdot \frac{\eta_g \cdot i_g}{R \cdot k_V} \cdot n_g$$

$$\frac{dP_{mech}}{dn_g} = -\frac{2 \cdot \eta_g \cdot i_g^2}{R \cdot k_V^2} \cdot n_g + \left(U_b - R \cdot I_{0m}\right) \cdot \frac{\eta_g \cdot i_g}{R \cdot k_V} = 0 \qquad n_{g\,Pmax} = \frac{U_b - R \cdot I_{0m}}{2} \cdot \frac{k_V}{i_g} = \frac{n_0}{2}$$

In the equation for mechanical power, drive speed is substituted with the equation for drive speed at maximum power. That gives the *value* of maximum mechanical power:

$$P_{mech\,max} = \frac{\left(U_b - R \cdot I_{0m}\right)^2}{4 \cdot R} \cdot \eta_g$$

Because P_{mech} is an inverted parabola, drive efficiency η_d is one as well, just skewed by the inversely proportional P_{el} line, so there is a maximum as well. This one is significantly harder to derive as equation. That is why we spared us substituting the P terms with more complicated expressions dependent on drive rotational speed n_g. The usual trick is using expressions dependent on current I and finally substituting this with n_g. So first we make the two powers and their components depend on current.

For that we need an equation giving rotational speed n_g dependent on current I. We equate two equations from the section Electrical-Mechanical Conversion and substitute with one from Mechanical-Mechanical Conversion to derive this equation:

$$U_{mC} = U_b - R \cdot I \quad \text{and} \quad U_{mC} = \frac{n_m}{k_V} \quad \text{and} \quad n_g = \frac{n_m}{i_g} \quad \text{make} \quad n_g = (U_b - R \cdot I) \cdot \frac{k_V}{i_g}$$

And we need an equation giving motor moment M_m dependent on current I. We use two equations from the Basic Solution chapter:

$$M_m = (I - I_{0m}) \cdot k_M \quad \text{and} \quad k_M = \frac{60}{2 \cdot \pi \cdot k_V} \quad \text{make} \quad M_m = (I - I_{0m}) \cdot \frac{60}{2 \cdot \pi \cdot k_V}$$

Now substituting M_m and n_g in the P_{mech} equation above gives the needed equation. It is just motor power output P_m expressed in electrical terms, being the proportion of current producing moment output times the proportion of voltage producing rotational speed. Gear efficiency reduces moment and hence also power output:

$$P_{mech} = M_m \cdot \eta_g \cdot i_g \cdot \frac{2 \cdot \pi}{60} \cdot n_g = (I - I_{0m}) \cdot (U_b - R \cdot I) \cdot \eta_g = P_m \cdot \eta_g$$

Electrical power input P_{el} depends on current I, anyway:

$$P_{el} = U_b \cdot I$$

The equation for P_{mech} above showed (again) that we can substitute it with motor power and this way get a simpler equation for drive efficiency. As desired, substituting the powers with the equations above makes for a manageable efficiency equation. Then we differentiate η_d with respect to I and equate the result with zero. That shows gear efficiency having no influence on *the point of* maximum drive efficiency:

$$\eta_d = \frac{P_{mech}}{P_{el}} = \frac{P_m}{P_{el}} \cdot \eta_g = \frac{(I - I_{0m}) \cdot (U_b - R \cdot I)}{U_b \cdot I} \cdot \eta_g = \left(1 - \frac{R \cdot I}{U_b} - \frac{I_{0m}}{I} + \frac{R \cdot I_{0m}}{U_b}\right) \cdot \eta_g$$

$$\frac{d\eta_d}{dI} = -\frac{R}{U_b} \cdot \eta_g + I_{0m} \cdot \eta_g \cdot \frac{1}{I^2} = 0 \quad \text{gives} \quad I_{\eta max} = \sqrt{\frac{U_b \cdot I_{0m}}{R}}$$

Using the drive speed n_g equation above and substituting current I with this square-root equation, finally results in the *point* (rotational speed) of maximum drive efficiency. In the drive efficiency equation, current I is substituted with the equation for current at maximum efficiency, giving maximum drive efficiency's *value*. Rearranging the equation after substitution is not simple, yet a quite short equation results:

$$n_{g\,\eta max} = \left(U_b - \sqrt{U_b \cdot R \cdot I_{0m}}\right) \cdot \frac{k_V}{i_g} \quad \text{is position and value is} \quad \eta_{d max} = \left(1 - \sqrt{\frac{R \cdot I_{0m}}{U_b}}\right)^2 \cdot \eta_g$$

See Feature Article by Joachim Bergmeyer (and his derivations).

Characteristic-Speed Ratios

Now that we have these characteristic speeds we can relate them to each other. There are no surprises, just a few insights that might be useful for assessing drives.

First, relating idle speed to "theoretical" or "ideal" maximum speed shows what both kinds of losses mean for a motor and for a motor-gear combination (drive) as well:

$$\frac{n_0}{n_{max}} = \frac{U_b - R \cdot I_{0m}}{U_b} = 1 - \frac{R \cdot I_{0m}}{U_b}$$

After all, system impedance R represents all electrical losses and idle current I_{0m} all mechanical losses. Of course this is simplified, and by definition there are no gear losses in this *second case* where gear friction is proportional to moment output, which is zero here.

Impedance R times idle current I_{0m} is the voltage drop when idling, and relating that to battery voltage U_b is the proportion of this battery voltage lost. That subtracted from 1 is the proportion of battery voltage remaining and seen by the motor coil. Now since voltage makes for rotational speed (proportionate to the k_V value), that is also the proportion of "theoretical" or "ideal" maximum speed remaining in reality as idle speed.

A "better" motor means less electrical and mechanical losses than those of a "cheap" motor. This is achieved for instance by using neodymium magnets instead of ferrite magnets and ball bearings instead of sleeve bearings (and better collector and brushes in case of a brushed motor). The better a motor is, the closer is its idle speed to the "theoretical" or *ideal* maximum speed.

So all losses in a drive result in more or less reduction of rotational speed. What we have seen for idle-speed so far will hold for other characteristic speeds as well. Now this idle-speed will be used as practical reference for more ratios, which will be just a bit more complicated though.

In the previous section, we had already seen that maximum mechanical power output $P_{mech\ max}$ is delivered at half idle-speed. The derivation is repeated here, just to show that the idle/ideal speed ratio is contained twice:

$$\frac{n_{Pmax}}{n_0} = \frac{\frac{U_b - R \cdot I_{0m}}{2}}{U_b - R \cdot I_{0m}} = \frac{\frac{1}{2} \cdot \left(1 - \frac{R \cdot I_{0m}}{U_b}\right)}{1 - \frac{R \cdot I_{0m}}{U_b}} = \frac{\frac{1}{2} \cdot \frac{n_0}{n_{max}}}{\frac{n_0}{n_{max}}} = \frac{1}{2}$$

Actually this is simple and general: Half idle-speed is the lowest speed that is reasonable by all means. It is because at lower speeds, the mechanical power output is lower while the electrical power input is even higher, and that is inefficient.

In practice, even this speed may be too low. Efficiency is not exactly good there, and that means much heat is produced in the motor. Depending on power setting (voltage) and heat removal (cooling), even short-time tolerable power may be much lower than maximum power and thus tolerable speed higher than maximum-power speed. In this case, half idle-speed is a "theoretical" lower limit, but it is still the *absolute* lower limit by all means.

The practical lower speed-limit stems from heat production, which in turn depends on power and efficiency. So we have to consider battery voltage, which defines power, and rotational speed, which defines efficiency. The ratio of maximum-efficiency speed and idle speed looks not too complicated:

$$\frac{n_{\eta max}}{n_0} = \frac{U_b - \sqrt{U_b \cdot R \cdot I_{0m}}}{U_b - R \cdot I_{0m}} = \frac{1 - \sqrt{\dfrac{R \cdot I_{0m}}{U_b}}}{1 - \dfrac{R \cdot I_{0m}}{U_b}} = \frac{\sqrt{\eta_{m\,max}}}{\dfrac{n_0}{n_{max}}}$$

Interestingly enough, the idle/ideal speed ratio seems to reappear here again twice, now just with a square-root of the proportion of speed lost (by impedance and friction) in the numerator. But the term in the numerator is actually the square root of maximum motor efficiency, which had been implicitly contained in the equation for maximum drive efficiency in the previous section.

Thus we can definitely conclude that maximum-efficiency speed is much closer to idle speed than to maximum-power speed (which is always half idle-speed). That means in turn that maximum efficiency is reached at high rotational speed where power is low, so high efficiency and high power are mutually exclusive.

For any given drive, there is quite a difference between the full-power and the cruise-power cases. Since battery voltage is seen as substantially lower in cruise flight, and since it is in the denominator in the equation above, efficiency is lower over the whole rotational-speed range, which is smaller as well. Also, a drive may be used with more or less battery cells and thus voltage, what would make efficiency somewhat higher or lower, respectively.

And there is some difference between "cheap" and "better" drives: The motors' peak efficiencies may be 0.74 or 0.85, respectively, what looks like quite far from idle speed. But their square roots would be bigger, 0.85 or 0.92, respectively. So both drives have their peak efficiencies close to idle speed, the "better" just even closer, at even less power than a "cheap" drive.

Then again, a "better" drive is more efficient and produces less heat than a "cheap" one. Hence its tolerable power is higher and its tolerable speed lower, that is closer to maximum-power speed. If full use is made of its power potential, the "better" drive is even further away from its peak efficiency than the "cheap" one. Still its efficiency at tolerable power is better.

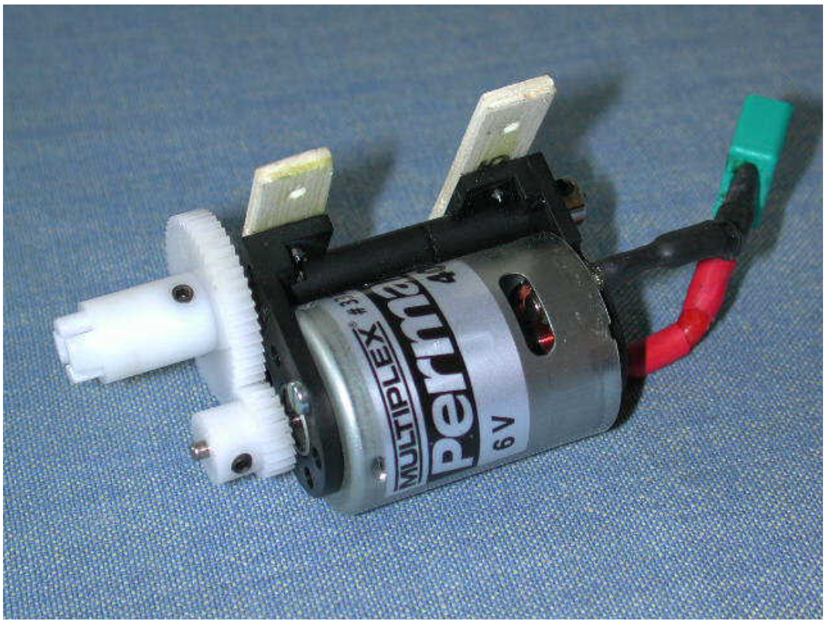
Given that modern motors – brushless, neodymium magnets, ball bearings – are all "better", and gears as well, this comparison is actually pointless nowadays. There is yet one insight that might be useful: Electric model-airplane drives are always operated quite far from their peak efficiency, that is at full-power setting.

In practical terms, they would reach their peak efficiency only at high dive speeds but never at ordinary flight speeds. The drive's peak efficiency may be even 5% lower than that of the motor alone, and it is fair to say that drive efficiency in operation is another 10% lower. Motor peak-efficiency is suitable as a *comparative* value, and that is why it may be specified for a motor. It may be used as an advertising point as well, but in any case we have to take it with a grain of salt or just as what it is, respectively.

Motor/Gear Example

Now that all necessary equations are at hand, illustrating diagrams shall show all interesting drive characteristics as lines or curves over a rotational-speed (n_g) axis. To this end, a real case has to be chosen as an example, which is as prototypical as possible. In a sense, a drive that is little short of vintage is just that:

It is a drive for a vintage-style parkflyer brought out in 2000. Parkflyers were a new category that made the hobby more affordable and practicable by means of a small and inexpensive electric drive. Characteristic were a 400-size brushed can-motor, a primitive reduction gear for a quite efficient slow-flight propeller, a simple "brushed" ESC, and a 7-cell NiCd battery to be charged from a car battery with a simple charger.



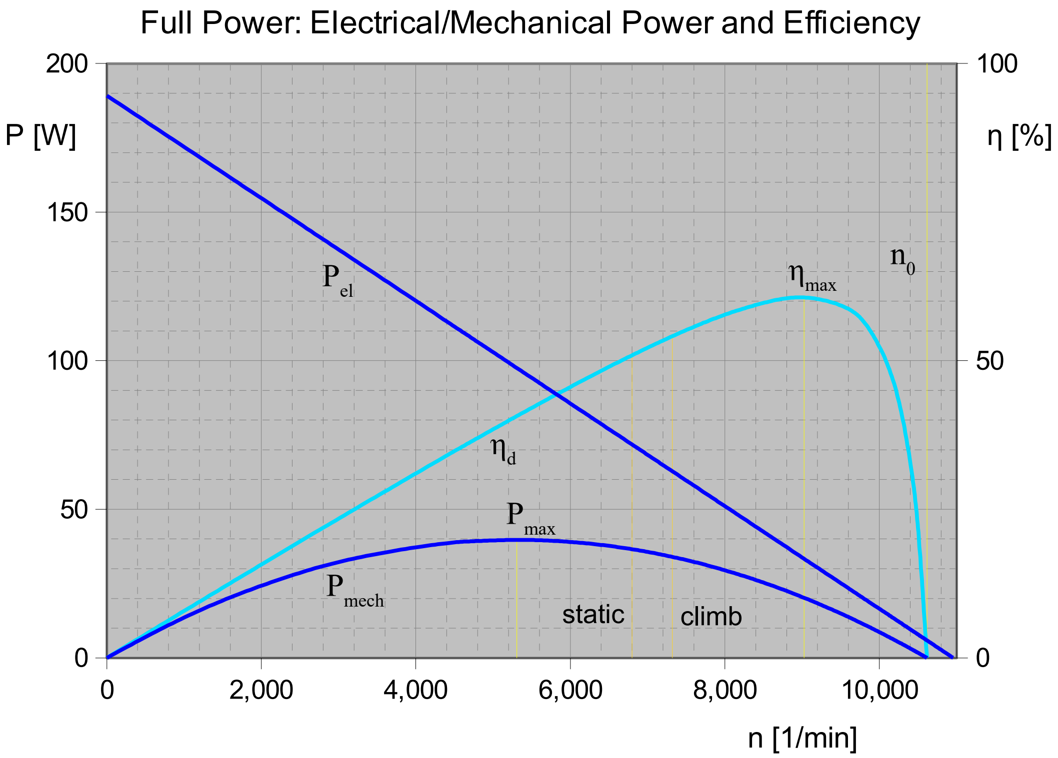
The 7x6.5" propeller was made by Günther, a German manufacturer of flying toys. Actually, this is a toy propeller as well as the gear may be seen as a toy gear. The can motors have been made in huge numbers for automotive applications. All that qualifies the drive as "cheap" in the sense of this chapter.

The calculations described here have been developed for this very drive in the first place. It was not yet customary back then to specify all necessary characteristics. They had to be collected from different sources and derived by own measurements or calculations, respectively. The result in this case is well-nigh typical again:

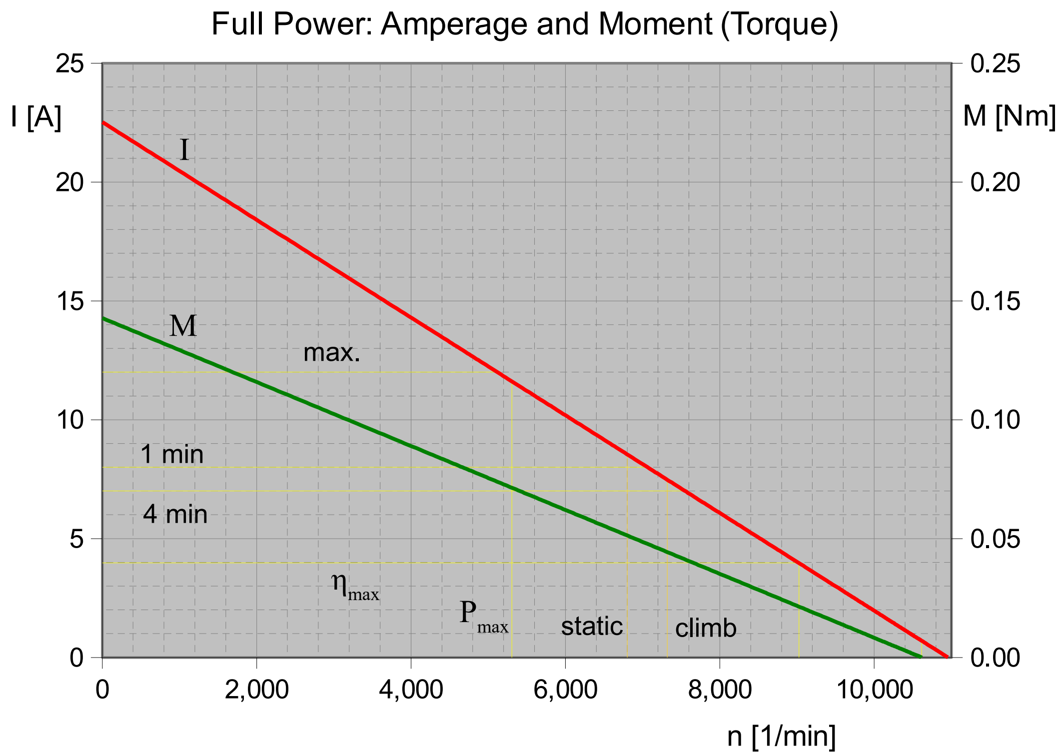
U_b	8.4 [V]	1.2 V nominal NiCd cell voltage, 7 cells
R	0.373 [Ω]	0.24 Ω motor (specified) + 0.133 Ω battery, ESC ("tweaked")
I_{0m}	0.7 [A]	specified, actual value may differ
k_V	3000 [min^{-1}/V]	specified, actual value may differ
i_g	2.3 [-]	specified, actually 49:22=2.227
η_g	0.89 [-]	"tweaked" by experiment and measurement
I_{max}	12/8/7 [A]	absolute/1 minute/4 minutes, loosely specified

Motor/Gear Diagrams

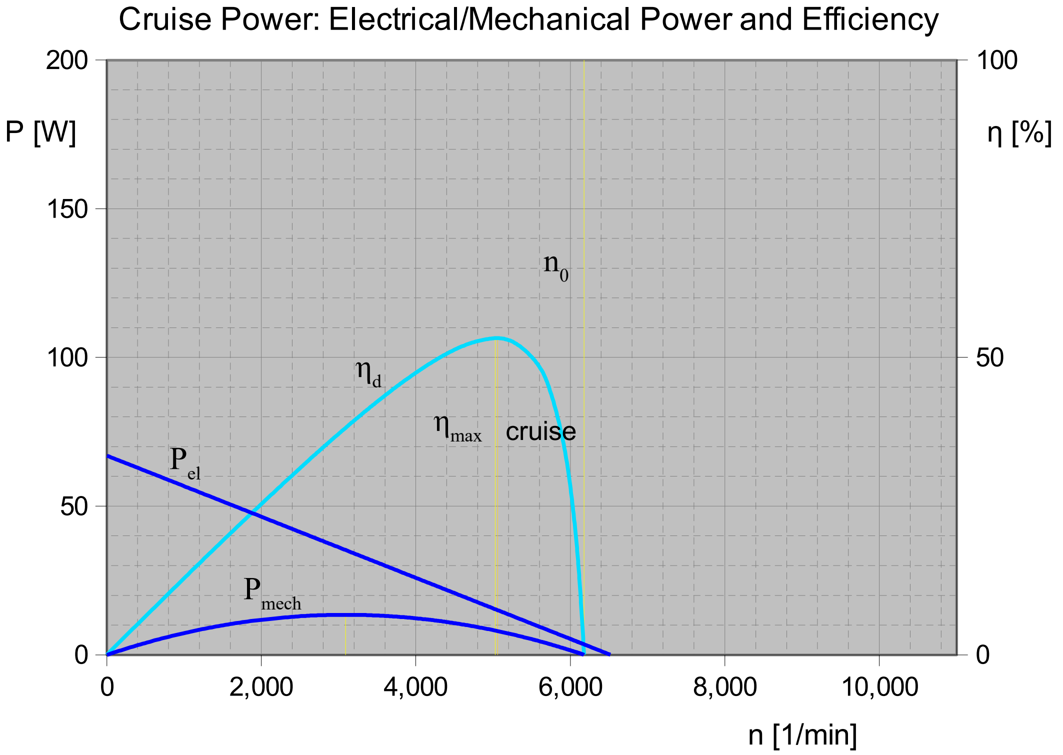
Overall drive efficiency is the ratio of mechanical and electrical power. Its peak value is even 61%, but it's only 54% in a climb (in this case of a retro-style parkflyer):



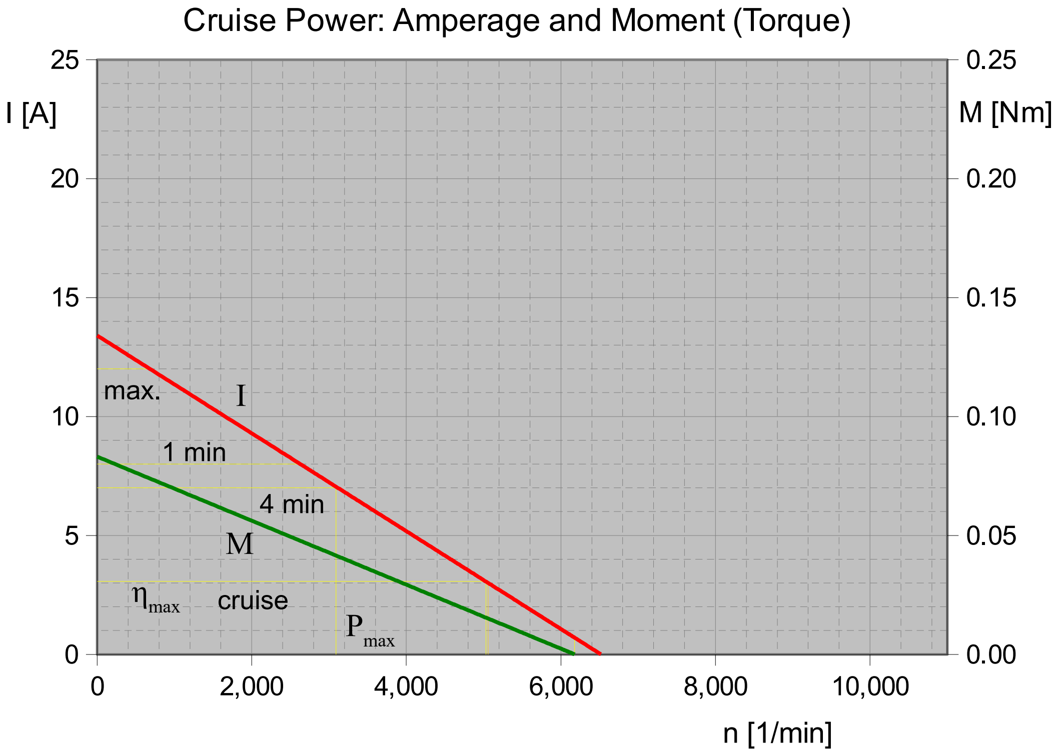
Coincidentally (in this case), maximum tolerable amperage is even at a slightly slower rotational speed than maximum power, but static run is slightly beyond the 1-minute amperage limit, and climb is slightly beyond the 4-minute amperage limit:



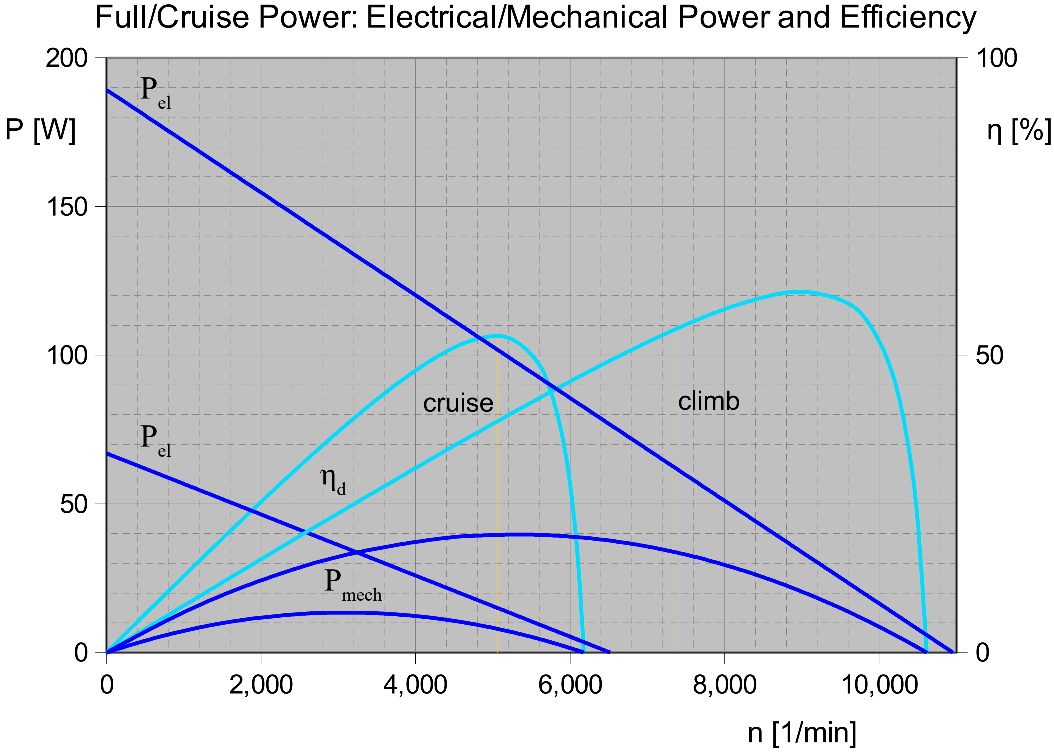
The diagrams on the previous page show the case of full-power, that is 8.4 V battery voltage. The following two diagrams show the case of cruise power (which is known from the performance calculations as well as the climb case). The ESC is set for an equivalent 0.6 voltage reduction factor, giving 5.0 V equivalent battery voltage:



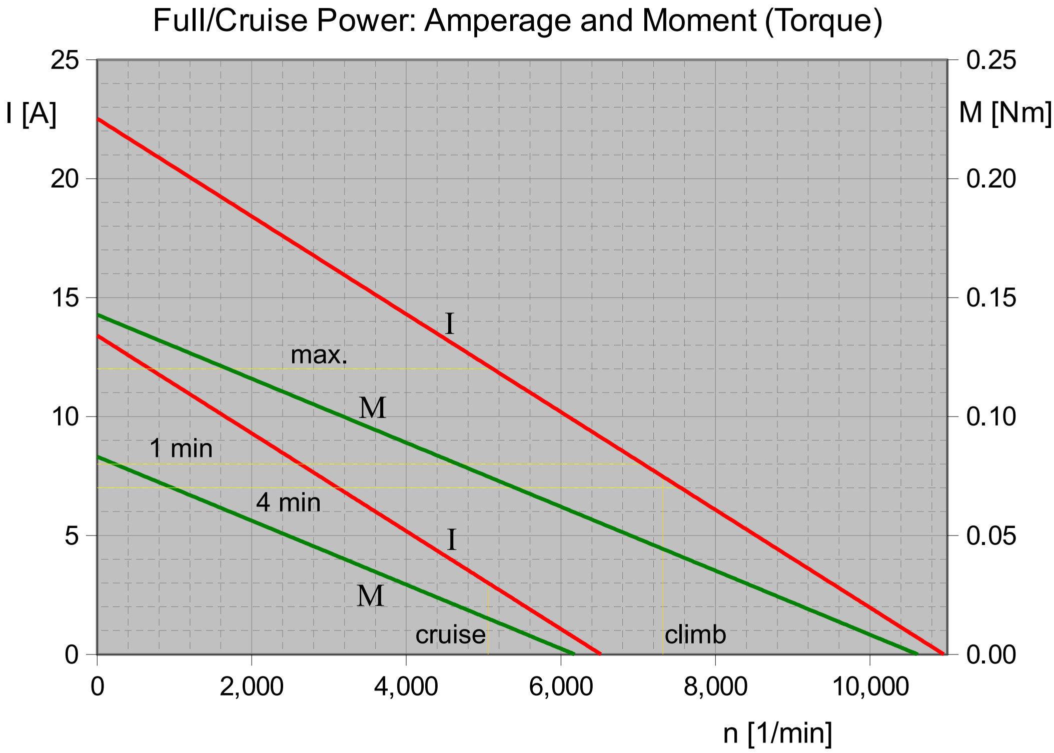
Cruise rotational speed advantageously coincides with maximum-efficiency rotational speed. That may be just a coincidence, but it might have been deliberate designing as well. The amperage limits still all apply, but they are not relevant in cruise flight:



Finally, we compare the full-power and cruise-power cases. The lines of electrical power are not parallel (just an observation), and the efficiency curves (particularly peak efficiencies) are different, both due to the different voltages. However, the efficiencies in cruise and climb are virtually equal (53% or 54%):

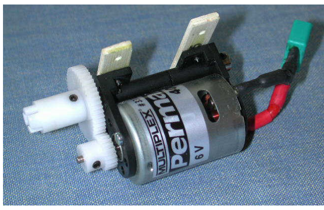


The lines of amperage and moment (torque), respectively, *are* parallel. That means the drive's "stiffness" (it's decrease of rotational speed with increase of load) does not depend on power setting. Cruise and climb currents (3.0 A, 7.5 A) are worth noting:



Motor/Gear Comparison

There are no hard and fast rules about drawing conclusions from drive characteristics, yet there are striking similarities in different cases. To give a clue, the vintage "cheap" example drive is compared to two rather different ones. The first comparative example is vintage as well, just a "better" or premium "brushed" inrunner drive. The second is ten years younger and nowadays typical with brushless/gearless outrunner motor and a LiPo battery. Efficiencies and a few ratios are worth noting:



Type of model	55" retro parkflyer	100" thermal glider	95" Sr. Telemaster
Weight of model	0.85kg / 1.9lbs	1.7kg / 3.75lbs	4.5kg / 10lbs
Motor	400-size "can"	480-size premium	4130 brushless
Gear i_g – η_g	2.3:1 – 89%	4.4:1 – 95%	no gear – 100%
Weight of drive	95g / 3.35oz	184g / 6.5 oz	405g / 14.3oz
Power "in" static	70 W (82 W/kg)	150 W (88 W/kg)	500 W (111 W/kg)
Power "out" static	37 W (0.39 W/g)	100 W (0.54 W/g)	350 W (0.86 W/g)
Battery (weight)	7s 1000 NiCd (170g)	7s 2300 NiCd (442g)	4s 5000 LiPo (548g)
B. Voltage (energy)	8.4 V (49 Wh/kg)	8.4 V (44 Wh/kg)	14.8 V (135 Wh/kg)
Motor / Drive k_V	3000 / 1300 rpm/V	3440 / 780 rpm/V	360 / 360 rpm/V
Idle/max. I_{0m} / I_{max}	0.7 A / 8 A 1min	0.76 A / 20 A 1 min	1.3 A / 60 A 1 min
Motor / total R	0.24 / 0.373 Ω	0.071 / 0.134 Ω	0.062 / 0.117 Ω
Peak eff. $\eta_{d\,max}$ ($\eta_{m\,max}$)	61% (74%)	75% (85%)	81% (86%)
Cruise/climb eff.	53% / 54%	66% / 65%	73% / 71%
Cruise/climb amps	3.0A / 7.5A = 0.40	3.7A / 16.9A = 0.22	9.2A / 33.6A = 0.27
Cruise/climb rpm	5050 / 7250 = 0.70	2400 / 4800 = 0.50	2150 / 3900 = 0.55
Climb/ideal rpm	7250/10920 = 0.66	4800 / 6550 = 0.73	3900 / 5330 = 0.73

The first drive is so weak and inefficient that its amperage in cruise has to be even 40% of that in climb. Now cruise rpm is even 70% of climb rpm, and climb rpm is only 66% of ideal rpm. In more "normal" cases like the two other drives, about ¼, ½, and ¾, respectively, would be good first-order estimates for these ratios.

The "better" the drive the better are all its efficiencies and the lesser is the difference between motor and drive peak-efficiency. The respective efficiencies in cruise and climb are about equal, and up to ten percent-steps lower than peak efficiency. There are size effects, but they are small. A brushless/gearless replacement for the small first drive would have two percent-steps less peak efficiency than the big third drive.

To be continued...

Workflow

Spreadsheet Sequence

There are three modules of calculation, each in one of three interlinked spreadsheets:

1. Electrical characteristics of motor, speed controller, battery, cables, and plugs; as well as mechanical characteristics of motor and gear.
2. Aerodynamic and mechanical characteristics of the propeller.
3. Characteristics of the whole drive dependent on flight speed.

And there are another three, optional interlinked modules/spreadsheets:

4. Aerodynamic characteristics (coefficients) of the wing airfoil.
5. Aerodynamic characteristics of the whole airframe.
6. Performance characteristics of the whole airplane dependent on flight speed.

The former suffice to find a suitable drive or to compare different drives for a model. The drive characteristics are calculated for two cases: full power and cruise power. The latter not only show the performance characteristics of the whole airplane but thereby also help finding a suitable cruise power for the former.

It's typical to begin a complete calculation with nominal values specified by manufacturers or even with estimates and then gradually refine it by tweaking parameters until it is consistent. Basically, it can be calibrated with measured values. These may be published by component manufacturers, so the calculation is calibrated to the specific *type* of component. If the drive is already at hand, it can be measured for a calibration to the specific component *samples*, at first static for a safe maiden flight and finally in-flight just to be sure.

The spreadsheets are in one single file in the Microsoft® Excel® XLSX format which should work in OpenOffice™/LibreOffice Calc as well (which has its own ODS format).

Example

There is a prototypical example, that is the drive of the author's Senior Telemaster *Plus* model airplane. The calculation spreadsheets are available for download from the author's Web site.

There is also a comprehensive explanation of the calculation results – both flight performance and drive characteristics – on the author's review Web page for the model.

To be continued...